Roll No. ☐☐☐☐☐☐☐☐

Q.P. Code 65/1/1

MATHEMATICS

CBSE Class XII Board Examination 2026 — with answers

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. The paper has 38 questions in five sections (A to E); all of them must be answered.
2. Section A: Q1–18 are multiple choice and Q19–20 are Assertion-Reason questions, 1 mark each.
3. Section B: Q21–25, very short answer (VSA), 2 marks each.
4. Section C: Q26–31, short answer (SA), 3 marks each.
5. Section D: Q32–35, long answer (LA), 5 marks each.
6. Section E: Q36–38, case-study based, 4 marks each.
7. There is no overall choice, but some questions in Sections B to E offer an internal choice (OR). Calculators are not allowed.

SECTION A

Q1–20 carry 1 mark each (MCQ; Q19–20 are Assertion-Reason).

1. Given $2\cos^{-1} x = y$, the values that y can take lie in **1**

(A) $0 \leq y \leq \pi$ **(B)** $-\pi \leq y \leq \pi$

(C) $0 \leq y \leq 2\pi$ **(D)** $-\pi \leq y \leq 0$

ANSWER
(C) $0 \leq y \leq 2\pi$

Principal value branch of $\cos^{-1}x$ is $[0, \pi]$, so $0 \leq \cos^{-1}x \leq \pi$.
Multiply by 2: $0 \leq 2\cos^{-1}x \leq 2\pi$, i.e. $0 \leq y \leq 2\pi$.
Common mistake: answering $[0, \pi]$ – that is the range of $\cos^{-1}x$ itself, not of $2\cos^{-1}x$.

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2. Which one of these can NOT be the order of a row matrix? **1**

(A) 2×1 **(B)** 1×2

(C) 1×1 **(D)** $1 \times n$

ANSWER
(A) 2×1

A row matrix has exactly one row, so its order is $1 \times n$.
1×2, 1×1 and $1 \times n$ all have one row; 2×1 has two rows (it is a column matrix).

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3. For matrices A and B of suitable orders, which of these properties hold? **1**

(i) $(A + B)' = A' + B'$
(ii) $(A - B)' = B' - A'$
(iii) $(AB)' = A'B'$
(iv) $(kAB)' = kB'A'$, k a scalar

(A) (i) only **(B)** (i), (ii) and (iii)

(C) (i) and (ii) **(D)** **(i) and (iv)**

ANSWER
(D) (i) and (iv)

(i) Transpose of a sum is the sum of transposes – true.
(ii) $(A - B)' = A' - B'$, not $B' - A'$ – false.
(iii) Reversal law: $(AB)' = B'A'$, not $A'B'$ – false.
(iv) $(kAB)' = k(AB)' = kB'A'$ – true.
So only (i) and (iv) are correct.

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4. Let $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$. Then **1**

(A) $\boldsymbol{\Delta_1 = 2\Delta_2}$ **(B)** $\boldsymbol{\Delta_2 = -2\Delta_1}$

(C) $\boldsymbol{\Delta_1 = \Delta_2}$ **(D)** $\boldsymbol{\Delta_2 = -\Delta_1}$

ANSWER

(B) $\Delta_2 = -2\Delta_1$

Δ_1 is a diagonal determinant: $\Delta_1 = 1 \times 2 \times 3 = 6$.

Δ_2: expand along R_3 (only the 6 is non-zero): $\Delta_2 = 6 \times (0 \cdot 0 - 2 \cdot 1) = 6 \times (-2) = -12$.

So $\Delta_2 = -12 = -2 \times 6 = -2\Delta_1$.

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5. One value of x satisfying $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is **1**

(A) 0 **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

ANSWER

(B) $x = \pi/4$

Determinant = $\cos x \cdot \sin x - (\sin x)(-\cos x) = 2 \sin x \cos x = \sin 2x$.

$\sin 2x = 1 \Rightarrow 2x = \pi/2 \Rightarrow x = \pi/4$.

Check others: $x = 0$ gives 0, $x = \pi/3$ gives $\sqrt{3}/2$, $x = \pi/2$ gives 0.

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6. A and B are skew-symmetric matrices of the same order. Which of the following is also skew-symmetric? **1**

(A) AB **(B)** AB + BA

(C) $\boldsymbol{(A+B)^2}$ **(D)** **A − B**

ANSWER

(D) A − B

Given $A' = -A$, $B' = -B$.

$(A - B)' = A' - B' = -A + B = -(A - B) \Rightarrow A - B$ is skew-symmetric.

For the others: $(AB)' = B'A' = BA$ (not $-AB$ in general); $(AB + BA)' = BA + AB$, symmetric;

$((A+B)^2)' = (A+B)^2$, symmetric.

7. On the interval [0, 3], the smallest value of $f(x) = x^3 - 12x$ is **1**

(A) −16 **(B)** −9

(C) 0 **(D)** 16

ANSWER
(A) −16

f′(x) = 3x² − 12 = 0 ⇒ x = ±2; only x = 2 lies in [0, 3].
Compare values at the critical point and the end points:
f(0) = 0, f(2) = 8 − 24 = −16, f(3) = 27 − 36 = −9.
Least value = −16 (at x = 2).

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8. If $\int \frac{3ax}{b^2+c^2x^2}\,dx = A\log|b^2 + c^2x^2| + K$, then A equals **1**

(A) 3a **(B)** $\frac{3a}{2b^2}$

(C) $\frac{3a}{b^2c^2}$ **(D)** $\frac{3a}{2c^2}$

ANSWER
(D) A = 3a/(2c²)

Put t = b² + c²x² ⇒ dt = 2c²x dx ⇒ x dx = dt/(2c²).
∫ 3a x dx/(b² + c²x²) = (3a/(2c²)) ∫ dt/t = (3a/(2c²)) log|b² + c²x²| + K.
So A = 3a/(2c²).

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9. Evaluate $\int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1}\,dx$. **1**

(A) 0 **(B)** log 2

(C) 2 log 2 **(D)** $\frac{1}{2}\log 2$

ANSWER
(A) 0

Let f(x) = x³/(x² + 2|x| + 1). Then f(−x) = −x³/(x² + 2|x| + 1) = −f(x), so f is odd.
For an odd function, ∫ from −a to a of f(x) dx = 0.
Tip: always test for odd/even before integrating over a symmetric interval.

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10. Find the area enclosed by the curve y = x|x|, the x-axis and the lines x = −1 and x = 1. **1**

(A) 0 **(B)** $\frac{1}{3}$

(C) $\frac{2}{3}$ **(D)** $\frac{4}{3}$

ANSWER
(C) 2/3 square units

y = x|x| equals x² for x ≥ 0 and −x² for x < 0; the curve is symmetric about the origin.
Area = ∫₋₁⁰ |−x²| dx + ∫₀¹ x² dx = 1/3 + 1/3 = 2/3.
Common mistake: ∫₋₁¹ x|x| dx = 0 is the signed integral, not the area.

11. For the differential equation $R\frac{dx}{dy} + Px = Q$, where P, Q, R are functions of y only, the integrating factor is **1**

(A) $e^{\int \frac{P}{Q} dy}$ **(B)** $e^{\int P\, dy}$

(C) $e^{\int \frac{P}{R} dy}$ **(D)** $e^{\int \frac{P}{R} dx}$

ANSWER
(C) e^(∫(P/R) dy)

Divide by R to get standard form: dx/dy + (P/R)x = Q/R.
This is linear in x with coefficient P/R (a function of y), so IF = e^(∫(P/R) dy).
Mistake to avoid: forgetting to make the coefficient of dx/dy equal to 1.

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12. Give the order and the degree (in that order) of the differential equation $\frac{d}{dx}(e^y) = 0$. **1**

(A) 0, 1 **(B) 1, 1**

(C) 2, 1 **(D)** 1, not defined

ANSWER
(B) 1, 1

Carry out the differentiation: e^y · dy/dx = 0.
Highest derivative is dy/dx ⇒ order 1; it appears to power 1 and the equation is a polynomial in dy/dx ⇒ degree 1.

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13. For what value of p are the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ perpendicular? **1**

(A) 0 **(B)** 1

(C) $\frac{5}{2}$ **(D)** $-\frac{5}{2}$

ANSWER
(C) p = 5/2

Perpendicular ⇒ dot product = 0.
(1)(2) + (2)(−p) + (3)(1) = 0 ⇒ 5 − 2p = 0 ⇒ p = 5/2.

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14. Points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ lie on one line. Find m. **1**

(A) 8 **(B)** −8

(C) 2 **(D)** $\frac{5}{2}$

ANSWER
(A) m = 8

Let A(−1, −1, 2), B(2, m, 5), C(3, 11, 6).
AB = (3, m + 1, 3), AC = (4, 12, 4).
Collinear ⇒ AB ∥ AC ⇒ 3/4 = (m + 1)/12 = 3/4 ⇒ m + 1 = 9 ⇒ m = 8.

15. Given $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find $|\vec{a} \cdot \vec{b}|$. **1**

(A) $6\sqrt{3}$ **(B)** $8\sqrt{3}$

(C) $12\sqrt{3}$ **(D)** $3\sqrt{12}$

ANSWER
(C) 12√3

|a × b| = |a||b| sin θ ⇒ 12 = 24 sin θ ⇒ sin θ = 1/2 ⇒ |cos θ| = √3/2.
|a · b| = |a||b||cos θ| = 24 × √3/2 = 12√3.
Shortcut: |a·b|² + |a×b|² = |a|²|b|² ⇒ |a·b|² = 576 − 144 = 432 ⇒ 12√3.

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16. Find the length of the perpendicular from the point (2, 5, 7) to the line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$. **1**

(A) 2 **(B)** 5

(C) $\sqrt{74}$ **(D)** $\sqrt{78}$

ANSWER
(C) √74

Direction ratios 1, 0, 0 through the origin ⇒ the line is the x-axis.
Foot of perpendicular from (2, 5, 7) is (2, 0, 0).
Distance = √(5² + 7²) = √(25 + 49) = √74.

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17. An LPP has objective function Z = 5x + 7y. Its feasible region (shown in a figure) is the quadrilateral with corners O(0, 0), (0, 2), (3, 4) and (7, 0). Find (maximum value of Z) − (minimum value of Z). **1**

(A) 8 **(B)** 29

(C) 35 **(D)** 43

ANSWER
(D) 43

Evaluate Z at each corner:
(0, 0): 0; (0, 2): 14; (3, 4): 15 + 28 = 43; (7, 0): 35.
Max = 43, min = 0 ⇒ difference = 43.

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18. What is the degree of the objective function of a linear programming problem? **1**

(A) 0 **(B)** 1

(C) 2 **(D)** Any natural number

ANSWER
(B) 1

In an LPP the objective function Z = ax + by is linear, i.e. a polynomial of degree 1 in the decision variables.

19. Assertion (A): When a fair die is thrown, the probability of getting a prime number, given that the number shown is odd, is $\frac{2}{3}$. **1**

Reason (R): For any two events A and B, $P(A|B) = \frac{P(A \cup B)}{P(B)}$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

ANSWER

(C) A true, R false

Odd outcomes B = {1, 3, 5}; primes among them = {3, 5}.

P(prime | odd) = 2/3 ⇒ A is true.

The correct formula is P(A|B) = P(A ∩ B)/P(B), not P(A ∪ B)/P(B) ⇒ R is false.

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20. Assertion (A): The lines x = py + q, z = ry + s and x = p′y + q′, z = r′y + s′ are perpendicular when pp′ + rr′ = 1. **1**

Reason (R): The lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

ANSWER

(D) A false, R true

First line: (x − q)/p = y/1 = (z − s)/r ⇒ direction ratios p, 1, r. Second line: p′, 1, r′.

Perpendicular ⇔ pp′ + 1·1 + rr′ = 0 ⇔ pp′ + rr′ = −1, not 1 ⇒ A is false.

R is the standard condition for perpendicular lines ⇒ true.

SECTION B

Q21–25 are very short answer (VSA) questions of 2 marks each.

21. A function is defined by $f(x) = \frac{|x-3|}{2(x-3)}$ for x < 3 and $f(x) = \frac{x-6}{6}$ for x ≥ 3. Is f continuous at x = 3? **2**

ANSWER

Yes – LHL = RHL = f(3) = −1/2, so f is continuous at x = 3

For x < 3, |x − 3| = −(x − 3), so f(x) = −(x − 3)/(2(x − 3)) = −1/2.
LHL = lim(x→3⁻) (−1/2) = −1/2.
RHL = lim(x→3⁺) (x − 6)/6 = −3/6 = −1/2.
f(3) = (3 − 6)/6 = −1/2.
LHL = RHL = f(3) ⇒ f is continuous at x = 3.

OR

Given $\sqrt{3}\,(x^2 + y^2) = 4xy$, find $\frac{dy}{dx}$ at the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

ANSWER

dy/dx = √3

Differentiate both sides w.r.t. x: √3(2x + 2y y′) = 4(y + x y′).
Collect y′: y′(2√3 y − 4x) = 4y − 2√3 x ⇒ y′ = (2y − √3x)/(√3y − 2x).
At (1/2, √3/2): numerator = √3 − √3/2 = √3/2; denominator = 3/2 − 1 = 1/2.
y′ = (√3/2)/(1/2) = √3.

22. A room-freshener bottle is an inverted cone with semi-vertical angle $\frac{\pi}{6}$. The volume of perfume in it falls steadily at 1 mm³/min. How fast is the perfume level falling when the level (height of liquid) is 10 mm? **2**

ANSWER

Level falls at 3/(100π) mm/min

Let h = height and r = radius of the liquid surface. tan(π/6) = r/h ⇒ r = h/√3.
V = (1/3)πr²h = (1/3)π(h²/3)h = πh³/9.
dV/dt = (π/3)h² dh/dt.
Given dV/dt = −1: dh/dt = −3/(πh²).
At h = 10: dh/dt = −3/(100π) mm/min.
So the level drops at 3/(100π) mm/min (the minus sign shows a decrease).

23. P and Q are the points (1, 3, 2) and (−1, 0, 8). Find the vector of magnitude 14 along $\overrightarrow{QP}$. **2**

ANSWER

$4\hat{i} + 6\hat{j} - 12\hat{k}$

QP = P − Q = (1 + 1)î + (3 − 0)ĵ + (2 − 8)k̂ = 2î + 3ĵ − 6k̂.

|QP| = √(4 + 9 + 36) = 7.

Unit vector = (2î + 3ĵ − 6k̂)/7.

Required vector = 14 × unit vector = 4î + 6ĵ − 12k̂.

Careful: QP means from Q to P, i.e. P − Q.

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24. The vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ are adjacent sides of a parallelogram. Find vectors along its diagonals and their lengths. **2**

ANSWER

Diagonals $4\hat{i} - 2\hat{j} + 4\hat{k}$ (length 6) and $2\hat{i} - 2\hat{j}$ (length $2\sqrt{2}$)

Diagonals of a parallelogram with sides a, b are a + b and a − b.

a + b = 4î − 2ĵ + 4k̂, length √(16 + 4 + 16) = 6.

a − b = 2î − 2ĵ + 0k̂, length √(4 + 4) = 2√2.

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25. Write in simplest form: $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, where $0 < x < \frac{\pi}{4}$. **2**

ANSWER

π/4 − 2x

Divide numerator and denominator by cos 2x: (1 − tan 2x)/(1 + tan 2x) = tan(π/4 − 2x).

For 0 < x < π/4, π/4 − 2x lies in (−π/4, π/4), inside the principal range (−π/2, π/2).

So $\tan^{-1}[\tan(\pi/4 - 2x)] = \pi/4 - 2x$.

OR

Find the value of $\tan\left(\sin^{-1} 1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$.

ANSWER

$-1/\sqrt{3}$

$\sin^{-1}1 = \pi/2$.

$\cos^{-1}(-1/2) = \pi - \pi/3 = 2\pi/3$ (principal range [0, π]).

$\tan(\pi/2 - 2\pi/3) = \tan(-\pi/6) = -1/\sqrt{3}$.

Common mistake: taking $\cos^{-1}(-1/2) = -\pi/3$, which is outside [0, π].

SECTION C

Q26–31 are short answer (SA) questions of 3 marks each.

26. Evaluate $\int_0^1 x \tan^{-1} x \, dx$. **3**

ANSWER

$\pi/4 - 1/2$

Integrate by parts with u = $\tan^{-1}x$, dv = x dx:
$\int_0^1 x \tan^{-1}x \, dx = [(x^2/2)\tan^{-1}x]_0^1 - \int_0^1 (x^2/2)\cdot 1/(1+x^2)\, dx$
$= \pi/8 - (1/2)\int_0^1 [1 - 1/(1+x^2)]\, dx$
$= \pi/8 - (1/2)[x - \tan^{-1}x]_0^1$
$= \pi/8 - (1/2)(1 - \pi/4) = \pi/8 - 1/2 + \pi/8 = \pi/4 - 1/2.$

27. Find $\int \sqrt{\frac{x+2}{x-2}}\, dx$. **3**

ANSWER

$\sqrt{(x^2-4)} + 2\log|x + \sqrt{(x^2-4)}| + C$

Multiply inside the root by (x + 2)/(x + 2): integrand = $(x+2)/\sqrt{(x^2-4)}$.
Split: $\int x/\sqrt{(x^2-4)}\, dx + 2\int dx/\sqrt{(x^2-4)}$.
First: put $t = x^2 - 4$, $dt = 2x\, dx \Rightarrow (1/2)\int t^{(-1/2)}\, dt = \sqrt{t} = \sqrt{(x^2-4)}$.
Second: standard form $\int dx/\sqrt{(x^2-a^2)} = \log|x + \sqrt{(x^2-a^2)}| \Rightarrow 2\log|x + \sqrt{(x^2-4)}|$.
Answer: $\sqrt{(x^2-4)} + 2\log|x + \sqrt{(x^2-4)}| + C$.

OR

Find $\int \frac{x^2}{(x^2+9)(x^2+16)}\, dx$.

ANSWER

$-(3/7)\tan^{-1}(x/3) + (4/7)\tan^{-1}(x/4) + C$

Put $x^2 = t$ only for the partial fractions: $t/((t+9)(t+16)) = A/(t+9) + B/(t+16)$.
$t = A(t+16) + B(t+9)$. $t = -9 \Rightarrow A = -9/7$; $t = -16 \Rightarrow B = 16/7$.
Integral $= -(9/7)\int dx/(x^2+9) + (16/7)\int dx/(x^2+16)$
$= -(9/7)(1/3)\tan^{-1}(x/3) + (16/7)(1/4)\tan^{-1}(x/4) + C$
$= -(3/7)\tan^{-1}(x/3) + (4/7)\tan^{-1}(x/4) + C.$

28. Let $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x|\, dx$. Show that $I_1 - 4I_2 = 0$. **3**

ANSWER

$I_1 = 1$, $I_2 = 1/4$, so $I_1 - 4I_2 = 0$

I_1: $1/(1+\cos 2x)$ is even, so $I_1 = 2\int_0^{(\pi/4)} dx/(2\cos^2 x) = \int_0^{(\pi/4)} \sec^2 x\, dx = [\tan x]_0^{(\pi/4)} = 1$.
I_2: $|x|$ is even, so $I_2 = 2\int_0^{(1/2)} x\, dx = [x^2]_0^{(1/2)} = 1/4$.
$I_1 - 4I_2 = 1 - 4(1/4) = 0$. Hence proved.

29. Solve (general solution): $x^2 \frac{dy}{dx} = x^2 + xy + y^2$. **3**

ANSWER

$\tan^{-1}(y/x) = \log|x| + C$

Divide by x^2: $dy/dx = 1 + y/x + (y/x)^2$ – homogeneous.

Put $y = vx \Rightarrow dy/dx = v + x\, dv/dx$.

$v + x\, dv/dx = 1 + v + v^2 \Rightarrow x\, dv/dx = 1 + v^2$.

Separate: $dv/(1 + v^2) = dx/x \Rightarrow \tan^{-1}v = \log|x| + C$.

Replace $v = y/x$: $\tan^{-1}(y/x) = \log|x| + C$.

OR

Find the particular solution of $xy \frac{dy}{dx} = (x + 2)(y + 2)$ given that $y(1) = -1$.

ANSWER

$y - 2\log|y + 2| = x + 2\log|x| - 2$

Separate variables: $y/(y + 2)\, dy = (x + 2)/x\, dx$.

Write $y/(y + 2) = 1 - 2/(y + 2)$ and $(x + 2)/x = 1 + 2/x$.

Integrate: $y - 2\log|y + 2| = x + 2\log|x| + C$.

Use $x = 1$, $y = -1$: $-1 - 2\log 1 = 1 + 2\log 1 + C \Rightarrow C = -2$.

Particular solution: $y - 2\log|y + 2| = x + 2\log|x| - 2$.

30. Solve graphically: minimise $Z = 13x - 15y$ subject to $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$. **3**

ANSWER

Minimum $Z = -30$ at $(0, 2)$

Draw $x + y = 7$ (through $(7, 0)$, $(0, 7)$) and $2x - 3y + 6 = 0$ (through $(0, 2)$, $(-3, 0)$).

Feasible region: below $x + y = 7$, on the origin side of $2x - 3y + 6 = 0$, in the first quadrant – bounded.

Intersection of the two lines: $y = 7 - x \Rightarrow 2x - 21 + 3x + 6 = 0 \Rightarrow x = 3$, $y = 4$.

Corners and Z: $(0, 0) \rightarrow 0$; $(7, 0) \rightarrow 91$; $(3, 4) \rightarrow 39 - 60 = -21$; $(0, 2) \rightarrow -30$.

Minimum $Z = -30$ at $x = 0$, $y = 2$.

31. Bag I has 3 red and 4 white balls; bag II has 8 red and 6 white balls. A die is rolled: if it shows a number less than 3, a ball is drawn from bag I, otherwise from bag II. Find the probability that the ball drawn is red. **3**

ANSWER

P(red) = 11/21

E_1: die shows 1 or 2 ⇒ $P(E_1) = 2/6 = 1/3$; E_2: die shows 3–6 ⇒ $P(E_2) = 4/6 = 2/3$.
A: red ball. $P(A|E_1) = 3/7$, $P(A|E_2) = 8/14 = 4/7$.
Total probability: $P(A) = (1/3)(3/7) + (2/3)(4/7) = 3/21 + 8/21 = 11/21$.

OR

For two events X and Y, the probability that at least one of them happens is a, and the probability that exactly one of them happens is b. Prove that $P(X') + P(Y') = 2 - 2a + b$.

ANSWER

$P(X') + P(Y') = 2 - 2a + b$ (proved)

At least one: $P(X \cup Y) = a$.
Exactly one: $P(X \cup Y) - P(X \cap Y) = b \Rightarrow P(X \cap Y) = a - b$.
$P(X') + P(Y') = 2 - [P(X) + P(Y)] = 2 - [P(X \cup Y) + P(X \cap Y)]$
$= 2 - [a + (a - b)] = 2 - 2a + b$. Hence proved.

SECTION D

Q32–35 are long answer (LA) questions of 5 marks each.

32. On the set Z of integers, a relation is defined by R = {(x, y) : |x − y| is divisible by a fixed prime p}. Determine whether R is an equivalence relation. **5**

ANSWER

Yes, R is reflexive, symmetric and transitive – an equivalence relation

Reflexive: for any x ∈ Z, |x − x| = 0 and 0 is divisible by p ⇒ (x, x) ∈ R.
Symmetric: if (x, y) ∈ R, p divides |x − y| = |y − x| ⇒ (y, x) ∈ R.
Transitive: if (x, y), (y, z) ∈ R then p | (x − y) and p | (y − z) ⇒ p | (x − y) + (y − z) = x − z ⇒ p divides |x − z| ⇒ (x, z) ∈ R.
All three hold, so R is an equivalence relation.

OR

A function $f : \mathbb{R} - \{\frac{3}{5}\} \to \mathbb{R} - \{\frac{3}{5}\}$ is given by $f(x) = \frac{3x+2}{5x-3}$. Prove that f is both one-one and onto.

ANSWER

f is one-one and onto (it is its own inverse: x = (3y + 2)/(5y − 3))

One-one: let $f(x_1) = f(x_2)$: $(3x_1 + 2)(5x_2 - 3) = (3x_2 + 2)(5x_1 - 3)$.
Expand: $15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 - 9x_2 + 10x_1 - 6 \Rightarrow 19x_2 = 19x_1 \Rightarrow x_1 = x_2$.
Onto: take any y ≠ 3/5. Solve y = (3x + 2)/(5x − 3): 5xy − 3y = 3x + 2 ⇒ x(5y − 3) = 3y + 2 ⇒ x = (3y + 2)/(5y − 3).
This x is real, and x ≠ 3/5 (x = 3/5 would give 15y + 10 = 15y − 9, i.e. 10 = −9, impossible), and f(x) = y.
So every element of the codomain has a pre-image ⇒ f is onto.

33. Let $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$. Find A^{-1} and use it to solve: −2y + z = 7, 2x − y − z = 8, x − 2y = 10. **5**

ANSWER

A^{-1} = [[2, 1, 3], [2, 1, 2], [−3, −2, −4]]; x = 0, y = −5, z = −3

|A| = 0(0 − 2) − 2(0 + 2) + 1(2 + 1) = −4 + 3 = −1 ≠ 0.

Cofactors: $C_{11} = -2$, $C_{12} = -2$, $C_{13} = 3$; $C_{21} = -1$, $C_{22} = -1$, $C_{23} = 2$; $C_{31} = -3$, $C_{32} = -2$, $C_{33} = 4$.

adj A = [[−2, −1, −3], [−2, −1, −2], [3, 2, 4]] ⇒ A^{-1} = adj A/|A| = [[2, 1, 3], [2, 1, 2], [−3, −2, −4]].

The coefficient matrix of the system has rows (0, −2, 1), (2, −1, −1), (1, −2, 0) – this is A′. So A′X = B with B = (7, 8, 10).

$X = (A')^{-1}B = (A^{-1})'B$ = [[2, 2, −3], [1, 1, −2], [3, 2, −4]](7, 8, 10).

x = 14 + 16 − 30 = 0; y = 7 + 8 − 20 = −5; z = 21 + 16 − 40 = −3.

Check: −2(−5) − 3 = 7 ✓, 0 + 5 + 3 = 8 ✓, 0 + 10 = 10 ✓.

Key point: notice the system uses the transpose of A.

OR

The matrix $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is singular. Find all x in $\left[0, \frac{\pi}{2}\right]$.

ANSWER

x = 0 or x = π/6

Singular ⇒ determinant = 0. Expand along R_1:

3(8 − 7cos 2x) + 1(−14 + 11cos 2x) + sin 3x(−49 + 44) = 0

⇒ 24 − 21cos 2x − 14 + 11cos 2x − 5 sin 3x = 0 ⇒ 10 − 10cos 2x − 5 sin 3x = 0 ⇒ 2cos 2x + sin 3x = 2.

Use cos 2x = 1 − 2sin²x and sin 3x = 3sin x − 4sin³x:

2 − 4sin²x + 3sin x − 4sin³x = 2 ⇒ sin x(4sin²x + 4sin x − 3) = 0 ⇒ sin x(2sin x − 1)(2sin x + 3) = 0.

sin x = 0 ⇒ x = 0; sin x = 1/2 ⇒ x = π/6; sin x = −3/2 impossible.

In [0, π/2]: x = 0, π/6.

34. Given x = cos t and y = cos mt, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$. **5**

ANSWER

(1 − x²)y″ − xy′ + m²y = 0 (proved)

dx/dt = −sin t, dy/dt = −m sin mt ⇒ dy/dx = m sin mt/sin t, i.e. sin t · dy/dx = m sin mt.

Differentiate w.r.t. x (using dt/dx = −1/sin t):

sin t · y″ + cos t · (dt/dx) · y′ = m² cos mt · (dt/dx).

Multiply by sin t: sin²t · y″ − cos t · y′ = −m² cos mt.

Substitute sin²t = 1 − x², cos t = x, cos mt = y:

(1 − x²)y″ − x y′ = −m²y ⇒ (1 − x²)y″ − x y′ + m²y = 0.

35. Are the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ parallel? If they are, find the distance between them; if not, find where they meet (if they intersect). **5**

ANSWER

Not parallel; they intersect at $(-1, -1, -1)$

Direction ratios: 2, 3, 4 and 5, 2, 1 – not proportional ⇒ not parallel.

General points: $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ and $(5\mu + 4, 2\mu + 1, \mu)$.

Equate: $2\lambda + 1 = 5\mu + 4$ and $3\lambda + 2 = 2\mu + 1 \Rightarrow 2\lambda - 5\mu = 3$, $3\lambda - 2\mu = -1$.

Solving: $\lambda = -1$, $\mu = -1$.

Third coordinate: $4(-1) + 3 = -1$ and $\mu = -1$ ✓ – consistent, so the lines intersect.

Point: $(2(-1) + 1, 3(-1) + 2, 4(-1) + 3) = (-1, -1, -1)$.

SECTION E

Q36–38 are case-study based questions of 4 marks each.

36. An online delivery company has 5000 subscribers, each paying ₹300 a year for unlimited free deliveries. It plans to raise the annual fee; for every ₹1 increase, 10 subscribers are expected to leave. Suppose the fee is raised by ₹x. **4**

(i) How many subscribers leave after a rise of ₹x? (1 mark)

(ii) Write the total revenue R(x) after the rise as a function of x. (1 mark)

(iii)(a) Find the value of x for which R(x) is maximum. (2 marks)

ANSWER

(i) 10x; (ii) $R(x) = (5000 - 10x)(300 + x)$; (iii) $x = 100$

(i) 10 subscribers per ₹1 $\Rightarrow$ 10x subscribers leave.

(ii) Remaining subscribers $5000 - 10x$, new fee $300 + x \Rightarrow R(x) = (5000 - 10x)(300 + x) = 1500000 + 2000x - 10x^2$.

(iii) $R'(x) = 2000 - 20x = 0 \Rightarrow x = 100$. $R''(x) = -20 < 0 \Rightarrow$ maximum at $x = 100$.

(Maximum revenue = 4900 × 400 = ₹19,60,000.)

OR

(iii) (b) Find the sub-intervals of (0, 5000) on which R(x) is increasing and on which it is decreasing.

ANSWER

Increasing on (0, 100), decreasing on (100, 5000)

$R(x) = (5000 - 10x)(300 + x) = 1500000 + 2000x - 10x^2$.

$R'(x) = 2000 - 20x = 20(100 - x)$; $R'(x) = 0$ at $x = 100$.

For $0 < x < 100$, $R'(x) > 0 \Rightarrow$ R increasing on (0, 100).

For $100 < x < 5000$, $R'(x) < 0 \Rightarrow$ R decreasing on (100, 5000).

37. An online jackpot has one first prize of ₹3,00,000, two second prizes of ₹2,00,000 each and three third prizes of ₹50,000 each. In all 1,00,000 tickets were sold at ₹100 each. Rohan buys one ticket. **4**

(i) What amounts can he possibly win? (1 mark)

(ii)(a) Find the probability that he wins at least ₹2,00,000. (2 marks)

(iii) Rohan also buys a ticket in another jackpot with a single prize of ₹5,00,000, where the chance of winning is 1 in 1,00,000. Find the probability that he wins on exactly one of his two tickets. (1 mark)

ANSWER

(i) ₹3,00,000, ₹2,00,000, ₹50,000 (or nothing); (ii) 3/100000; (iii) $699988/10^{10}$ = 174997/2500000000

(i) Possible winnings: ₹3,00,000, ₹2,00,000 or ₹50,000 (otherwise nothing).

(ii) Tickets worth at least ₹2,00,000 = 1 first + 2 second = 3 ⇒ P = 3/100000.

(iii) First jackpot: P(win) = 6/100000, P(lose) = 99994/100000. Second: P(win) = 1/100000, P(lose) = 99999/100000. The two are independent.

P(exactly one) = (6/100000)(99999/100000) + (99994/100000)(1/100000)

= (599994 + 99994)/10^{10} = $699988/10^{10}$ = 174997/2500000000.

OR

(ii) Find the probability that he does not win any amount.

(b)

ANSWER

99994/100000 = 49997/50000

Winning tickets = 1 + 2 + 3 = 6 out of 1,00,000.

P(no prize) = 1 − 6/100000 = 99994/100000 = 49997/50000.

38. A roundabout on a busy road has its outer boundary along the circle C_1: $x^2 + y^2 = 64$; a circular pond with a fountain at its centre is C_2: $x^2 + y^2 = 4$. **4**

(i) Draw C_1 and C_2 in one diagram. (1 mark)

(ii) Write y as a function of x for C_1 and for C_2. (1 mark)

(iii)(a) Using integration, find the area covered by the roundabout (region inside C_1). (2 marks)

ANSWER

(i) concentric circles at O, radii 8 and 2; (ii) $y = \pm\sqrt{(64 - x^2)}$, $y = \pm\sqrt{(4 - x^2)}$; (iii) 64π sq units

(i) Both circles have centre (0, 0); C_1 has radius 8 (cuts axes at ± 8), C_2 radius 2 (cuts axes at ± 2).
(ii) C_1: $y = \sqrt{(64 - x^2)}$ (upper half) or $y = -\sqrt{(64 - x^2)}$ (lower half); C_2: $y = \pm\sqrt{(4 - x^2)}$.
(iii) By symmetry, area $= 4\int_0^8 \sqrt{(64 - x^2)}\, dx = 4[(x/2)\sqrt{(64 - x^2)} + (64/2)\sin^{-1}(x/8)]_0^8 = 4[0 + 32 \cdot \pi/2]$ $= 64\pi$ sq units.

OR

(iii) Using integration, find the area of the circular pond.

(b)

ANSWER

4π sq units

Area of pond $= 4\int_0^2 \sqrt{(4 - x^2)}\, dx = 4[(x/2)\sqrt{(4 - x^2)} + 2\sin^{-1}(x/2)]_0^2 = 4[2 \cdot \pi/2] = 4\pi$ sq units.

— END —

Questions restated from the CBSE Class XII Board Examination 2026 (set 65/1/1) for practice. Reading passages and map items are described, not reproduced.