Roll No. ☐☐☐☐☐☐☐☐

Q.P. Code 65/1/1

MATHEMATICS

CBSE Class XII Board Examination 2025 — with answers

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. The paper has 38 questions in five sections (A to E); every question must be attempted.
2. Section A: Q1–18 are MCQs and Q19–20 are Assertion-Reason questions, 1 mark each.
3. Section B: Q21–25 are very short answer questions of 2 marks each.
4. Section C: Q26–31 are short answer questions of 3 marks each.
5. Section D: Q32–35 are long answer questions of 5 marks each.
6. Section E: Q36–38 are case-study questions of 4 marks each.
7. There is no overall choice; internal choices are given in 2 questions of Section B, 3 of Section C, 2 of Section D and 2 of Section E.
8. Calculators are not allowed.

SECTION A

20 multiple choice questions of 1 mark each (Q19–20 are Assertion-Reason).

1. For the diagonal matrix $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, which matrix is A^{-1}? **1**

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ANSWER

(D) $A^{-1} = A$ itself

The inverse of a diagonal matrix (no zero on the diagonal) is the diagonal matrix of reciprocals.

Reciprocals of −1, 1, 1 are −1, 1, 1.

So $A^{-1} = A$. Check: $A \cdot A = I$ because $(-1)(-1) = 1$.

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2. Given $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, which statement is true? **1**

(A) $\vec{a} \parallel \vec{b}$

(B) $\vec{a} \perp \vec{b}$

(C) $|\vec{b}| > |\vec{a}|$

(D) $|\vec{a}| = |\vec{b}|$

ANSWER

(B) a ⊥ b

$\vec{a} \cdot \vec{b} = 3(1) + 2(-1) + (-1)(1) = 3 - 2 - 1 = 0$.

Zero dot product (both vectors non-zero) means the vectors are perpendicular.

Also |a| = √14 and |b| = √3, so (C) and (D) are false; components are not proportional, so (A) is false.

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3. Evaluate $\int_{-1}^{1} \frac{|x|}{x}\, dx$ (with $x \neq 0$). **1**

(A) −1

(B) 0

(C) 1

(D) 2

ANSWER

(B) 0

$|x|/x = -1$ for x < 0 and $+1$ for x > 0.

Integral = $\int_{-1}^{0} (-1)dx + \int_{0}^{1} 1\, dx = -1 + 1 = 0$.

(Equivalently, |x|/x is an odd function, so its integral over [−1, 1] is 0.)

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4. Which one of these functions of x and y is NOT homogeneous? **1**

(A) $y^2 - xy$ **(B)** $x - 3y$

(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ **(D)** $\tan x - \sec y$

ANSWER

(D) tan x − sec y

F is homogeneous of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$.

(A) degree 2, (B) degree 1, (C) depends only on y/x, so degree 0.

(D) $\tan(\lambda x) - \sec(\lambda y)$ cannot be written as $\lambda^n(\tan x - \sec y)$, so it is not homogeneous.

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5. Let $f(x) = |x| + |x - 1|$. What can be said about f at x = 0 and at x = 1? **1**

(A) Continuous and differentiable at both points

(B) Differentiable but not continuous at both points

(C) Continuous but not differentiable at both points

(D) Neither continuous nor differentiable at both points

ANSWER

(C) continuous but not differentiable at x = 0 and x = 1

Sum of modulus functions is continuous everywhere.

f(x) = 1 − 2x for x < 0, = 1 for 0 ≤ x ≤ 1, = 2x − 1 for x > 1.

At x = 0: LHD = −2, RHD = 0 → not differentiable.

At x = 1: LHD = 0, RHD = 2 → not differentiable (sharp corners).

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6. A is a 2 × 2 matrix with det A = 4. Find det(4 adj A). **1**

(A) 16 **(B) 64**

(C) 256 **(D)** 512

ANSWER

(B) 64

For order n: $|kB| = k^n|B|$ and $|\text{adj } A| = |A|^{n-1}$.

Here n = 2: |adj A| = 4^1 = 4.

|4 adj A| = $4^2 \times 4 = 16 \times 4 = 64$.

Common mistake: writing 4 × 4 = 16 (forgetting the power n on the scalar).

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7. E and F are independent events with $P(E) = \frac{2}{3}$ and $P(F) = \frac{3}{7}$. Find $P(E \mid \bar{F})$. **1**

(A) $\frac{1}{6}$ **(B)** $\frac{1}{2}$

(C) $\frac{2}{3}$ **(D)** $\frac{7}{9}$

ANSWER

(C) 2/3

If E and F are independent, then E and $\bar{F}$ are also independent.

So $P(E \mid \bar{F}) = P(E) = \frac{2}{3}$.

Check: $P(E \cap \bar{F}) = \frac{2}{3} \cdot \frac{4}{7} = \frac{8}{21}$, divided by $P(\bar{F}) = \frac{4}{7}$ gives 2/3.

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8. Find the absolute maximum value of $f(x) = x^3 - 3x + 2$ on the interval [0, 2]. **1**

(A) 0 **(B)** 2

(C) 4 **(D)** 5

ANSWER

(C) 4

f'(x) = 3x² − 3 = 0 → x = 1 (x = −1 is outside [0, 2]).

f(0) = 2, f(1) = 1 − 3 + 2 = 0, f(2) = 8 − 6 + 2 = 4.

Largest value = 4 (at x = 2).

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9. A is a 3 × 3 matrix $\begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, B is the 3 × 1 column $\begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$ and C = [9 8 7] is a 1 × 3 row. Which products are defined? **1**

(A) AB only **(B)** AC only

(C) BA only **(D)** AB, AC and BA all

ANSWER

(A) only AB

A product PQ is defined when columns of P = rows of Q.

AB: (3×3)(3×1) → defined (3×1).

AC: (3×3)(1×3) → 3 ≠ 1, not defined.

BA: (3×1)(3×3) → 1 ≠ 3, not defined.

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10. If $\int \frac{2^{1/x}}{x^2}\,dx = k \cdot 2^{1/x} + C$, find k. **1**

(A) $\frac{-1}{\log 2}$ **(B)** $-\log 2$

(C) −1 **(D)** $\frac{1}{2}$

ANSWER

(A) k = −1/log 2

Put t = 1/x, so dt = −dx/x².

Integral = $-\int 2^t\,dt = -\frac{2^t}{\log 2} + C = -\frac{1}{\log 2}\,2^{1/x} + C$.

So k = −1/log 2.

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11. Vectors satisfy $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ with $|\vec{a}|=\sqrt{37}$, $|\vec{b}|=3$, $|\vec{c}|=4$. Find the angle between $\vec{b}$ and $\vec{c}$. **1**

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

ANSWER

(C) π/3

$\vec{b}+\vec{c}=-\vec{a}$, so $|\vec{b}+\vec{c}|^2=|\vec{a}|^2$.

9 + 16 + 2(3)(4)cos θ = 37 → 24 cos θ = 12 → cos θ = 1/2.

θ = π/3.

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12. Find the integrating factor of the differential equation $(x+2y^3)\frac{dy}{dx}=2y$. **1**

(A) $e^{y^2/2}$ **(B)** $\frac{1}{\sqrt{y}}$

(C) $\frac{1}{y^2}$ **(D)** e^{-1/y^2}

ANSWER

(B) 1/√y

Treat x as the dependent variable: $\frac{dx}{dy}=\frac{x+2y^3}{2y}$, i.e. $\frac{dx}{dy}-\frac{1}{2y}x=y^2$.

This is linear in x with P = −1/(2y).

IF = $e^{\int -\frac{1}{2y}dy}=e^{-\frac{1}{2}\log y}=y^{-1/2}=\frac{1}{\sqrt{y}}$.

Mistake to avoid: trying to make it linear in y – it is not.

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13. The matrix $A=\begin{bmatrix}7&0&x\\0&7&0\\0&0&y\end{bmatrix}$ is a scalar matrix. Find y^x. **1**

(A) 0 **(B)** 1

(C) 7 **(D)** ±7

ANSWER

(B) 1

Scalar matrix: all off-diagonal entries 0 and all diagonal entries equal.

So x = 0 and y = 7.

$y^x=7^0=1$.

14. A bounded feasible region of an LPP has corner points (2, 72), (15, 20) and (40, 15). For Z = 18x + 9y, which statement is correct? **1**

(A) Maximum at (2, 72), minimum at (15, 20)

(B) Maximum at (15, 20), minimum at (40, 15)

(C) Maximum at (40, 15), minimum at (15, 20)

(D) Maximum at (40, 15), minimum at (2, 72)

ANSWER

(C) max at (40, 15), min at (15, 20)

Z(2, 72) = 36 + 648 = 684.
Z(15, 20) = 270 + 180 = 450.
Z(40, 15) = 720 + 135 = 855.
Maximum 855 at (40, 15); minimum 450 at (15, 20).

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15. A and B are invertible matrices. Which of these statements is FALSE? **1**

(A) $(A+B)^{-1} = B^{-1} + A^{-1}$

(B) $(AB)^{-1} = B^{-1}A^{-1}$

(C) $\text{adj}(A) = |A|\,A^{-1}$

(D) $|A|^{-1} = |A^{-1}|$

ANSWER

(A) $(A + B)^{-1} = B^{-1} + A^{-1}$ is not correct

(B) is the reversal law – true.
(C) follows from $A^{-1} = \frac{1}{|A|}\text{adj } A$ – true.
(D) from $|A||A^{-1}| = |I| = 1$ – true.
(A) is false in general (e.g. A = B = I gives $(2I)^{-1} = \frac{1}{2}I$ but I + I = 2I). Also A + B may not even be invertible.

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16. An LPP with objective Z = ax + by has a bounded feasible region. Which is correct? **1**

(A) Z has only a maximum value

(B) Z has only a minimum value

(C) Z has both a maximum and a minimum value

(D) Z has neither a maximum nor a minimum value

ANSWER

(C) both maximum and minimum exist

Corner point theorem: on a bounded feasible region, the objective function has both a maximum and a minimum, and each occurs at a corner point.

17. The figure shows the parabola $y^2 = x$ with the region above the x-axis, below the upper half of the parabola and to the left of the line $x = 4$ shaded (from $x = 0$ to $x = 4$, y from 0 up to 2). Which integral gives the shaded area? **1**

(A) $\int_0^4 x\,dx$ **(B)** $\int_0^2 y^2\,dy$

(C) $2\int_0^4 \sqrt{x}\,dx$ **(D)** $\int_0^4 \sqrt{x}\,dx$

ANSWER

(D) $\int_0^4 \sqrt{x}\,dx$

Upper half of $y^2 = x$ is $y = \sqrt{x}$.
Vertical strips from $x = 0$ to $x = 4$, height $\sqrt{x}$: Area $= \int_0^4 \sqrt{x}\,dx = \frac{16}{3}$.
Option (C) would be the area on both sides of the x-axis; (B) is the area between the curve and the y-axis, not the shaded part.

18. The figure shows the graph of $y = \cos x$ for x from $-\pi/2$ to π (it rises to 1 at $x = 0$, crosses the x-axis at $\pm\pi/2$ and reaches -1 at π). Which graph represents its inverse? **1**

(A) an increasing curve on $(-\pi/2, \pi/2)$ with y from -1 to 1, through the origin;
(B) another increasing curve through the origin with y from -1 to 1;
(C) a decreasing curve with x from -1 to 1 and y from π (at $x = -1$) down to 0 (at $x = 1$), passing through $(0, \pi/2)$;
(D) a decreasing curve over all real x lying between the horizontal lines $y = 0$ and $y = \pi$, through $(0, \pi/2)$.

(A) Graph (A) **(B)** Graph (B)

(C) Graph (C) **(D)** Graph (D)

ANSWER

(C) the graph of $y = \cos^{-1}x$

The given curve is $y = \cos x$; its inverse uses the principal branch $[0, \pi]$.
$y = \cos^{-1}x$ has domain $[-1, 1]$, range $[0, \pi]$, is decreasing, with $\cos^{-1}(-1) = \pi$, $\cos^{-1}0 = \pi/2$, $\cos^{-1}1 = 0$.
That is exactly graph (C). Graph (D) is $\cot^{-1}x$ (domain all real numbers).

19. Assertion (A): The function $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = 3x - 5$ ($\mathbb{Z}$ = integers), is a bijection. **1**
Reason (R): A function is bijective when it is both one-one (injective) and onto (surjective).

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

ANSWER

(D) A is false, R is true

One-one: $3x_1 - 5 = 3x_2 - 5 \Rightarrow x_1 = x_2$, so f is injective.
Onto: for $y = 0$ we need $3x = 5$, $x = 5/3 \notin \mathbb{Z}$. So 0 has no pre-image; f is not onto.
Hence f is not bijective → A is false. R is the correct definition → true.

20. Assertion (A): The function $f(x) = 3x - 8$ for $x \leq 5$ and $f(x) = 2k$ for $x > 5$ is continuous at $x = 5$ when $k = 5/2$. **1**

Reason (R): f is continuous at x = a when the left-hand limit, the right-hand limit and f(a) are all equal.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

ANSWER

(D) A is false, R is true

LHL = f(5) = 3(5) − 8 = 7. RHL = 2k.

Continuity needs 2k = 7, so k = 7/2, not 5/2 → A is false.

R is the correct condition for continuity → true.

SECTION B

5 very short answer (VSA) questions of 2 marks each.

21. Find the derivative of $2^{\cos^2 x}$ with respect to $\cos^2 x$. **2**

ANSWER

$2^{\cos^2 x} \log 2$

Let u = $2^{\cos^2 x}$, v = $\cos^2 x$.

$\frac{du}{dx} = 2^{\cos^2 x} \log 2 \cdot (-2 \cos x \sin x)$.

$\frac{dv}{dx} = -2 \cos x \sin x$.

$\frac{du}{dv} = \frac{du/dx}{dv/dx} = 2^{\cos^2 x} \log 2$.

(Shortcut: with v = $\cos^2 x$, u = 2^v so du/dv = $2^v \log 2$.)

OR

Given $\tan^{-1}(x^2 + y^2) = a^2$ (a constant), find $\frac{dy}{dx}$.

ANSWER

dy/dx = −x/y

Take tan on both sides: $x^2 + y^2 = \tan(a^2)$, a constant.

Differentiate w.r.t. x: 2x + 2y dy/dx = 0.

dy/dx = −x/y.

22. Find the value of $\tan^{-1}\left[2 \sin\left(2 \cos^{-1} \frac{\sqrt{3}}{2}\right)\right]$. **2**

ANSWER

π/3

$\cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$ (principal value in [0, π]).

2 × π/6 = π/3; sin(π/3) = √3/2.

2 × √3/2 = √3.

$\tan^{-1} \sqrt{3} = \frac{\pi}{3}$.

23. The diagonals of a parallelogram are $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find its area. **2**

ANSWER

Area = √62/2 sq units

Area of parallelogram with diagonals $\vec{a}, \vec{b} = \frac{1}{2}|\vec{a} \times \vec{b}|$.

$\vec{a} \times \vec{b} = \hat{i}((-1)(-1) - (1)(3)) - \hat{j}((2)(-1) - (1)(1)) + \hat{k}((2)(3) - (-1)(1)) = -2\hat{i} + 3\hat{j} + 7\hat{k}$.

|a × b| = √(4 + 9 + 49) = √62.

Area = √62/2 square units.

Mistake to avoid: forgetting the ½ (that is only omitted when adjacent SIDES are given).

24. For $f(x) = 5x^{3/2} - 3x^{5/2}$ (x ≥ 0), find the intervals where f is (i) increasing (ii) decreasing. **2**

ANSWER

(i) increasing on [0, 1]; (ii) decreasing on [1, ∞)

$f'(x) = \frac{15}{2}x^{1/2} - \frac{15}{2}x^{3/2} = \frac{15}{2}\sqrt{x}(1-x)$.

f′(x) = 0 at x = 0, 1.

For 0 < x < 1: √x > 0 and 1 − x > 0, so f′ > 0 → increasing on [0, 1] (open interval (0, 1) also accepted).

For x > 1: 1 − x < 0, so f′ < 0 → decreasing on [1, ∞).

25. Two kite strings cross each other. They lie along $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ (strings are taut). Find the angle between the strings. **2**

ANSWER

θ = cos⁻¹(3/√21) = cos⁻¹(√21/7)

$\vec{a} \cdot \vec{b} = 6 - 2 + 8 = 12$.

|a| = √(9 + 1 + 4) = √14; |b| = √(4 + 4 + 16) = √24 = 2√6.

cos θ = 12/(√14 · 2√6) = 6/√84 = 3/√21.

θ = cos⁻¹(3/√21).

OR

For the points A(2, 1, 3) and B(8, −1, 0), find a vector of magnitude 21 units pointing opposite to $\overrightarrow{AB}$.

ANSWER

−18î + 6ĵ + 9k̂

$\overrightarrow{AB} = (8-2)\hat{i} + (-1-1)\hat{j} + (0-3)\hat{k} = 6\hat{i} - 2\hat{j} - 3\hat{k}$.

|AB| = √(36 + 4 + 9) = 7.

Unit vector opposite to AB = −(6î − 2ĵ − 3k̂)/7.

Required vector = 21 × that = −18î + 6ĵ + 9k̂.

SECTION C

6 short answer (SA) questions of 3 marks each.

26. Each side of an equilateral triangle grows at 3 cm/s. How fast is the area growing when a side is 15 cm? **3**

ANSWER

$dA/dt = 45\sqrt{3}/2$ cm²/s

Area $A = (\sqrt{3}/4)s^2$.

$dA/dt = (\sqrt{3}/4)(2s)\, ds/dt = (\sqrt{3}/2)\, s\, ds/dt$.

At $s = 15$, $ds/dt = 3$: $dA/dt = (\sqrt{3}/2)(15)(3) = 45\sqrt{3}/2$ cm²/s (≈ 38.97 cm²/s).

27. Solve graphically: maximise $Z = x + 2y$ subject to $x - y \geq 0$, $x - 2y \geq -2$, $x \geq 0$, $y \geq 0$. **3**

ANSWER

Feasible region is unbounded; Z has no maximum value

Draw lines $x = y$ and $x - 2y = -2$ (through (0, 1) and (−2, 0)).

Feasible region: below $y = x$, below $y = x/2 + 1$, in the first quadrant. The lines meet at (2, 2).

Region is unbounded with corner points O(0, 0) and (2, 2).

$Z(0, 0) = 0$, $Z(2, 2) = 6$.

Since the region is unbounded, test $x + 2y > 6$: e.g. (10, 0) lies in the region with $Z = 10 > 6$. The half-plane $x + 2y > 6$ meets the region.

Hence Z has no maximum value.

Mistake to avoid: declaring 6 as the maximum without checking the open half-plane.

28. Find $\int \frac{x+\sin x}{1+\cos x}\, dx$. **3**

ANSWER

$x \tan(x/2) + C$

Use $1 + \cos x = 2\cos^2(x/2)$ and $\sin x = 2 \sin(x/2)\cos(x/2)$.

Integrand $= \frac{x}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2}$.

By parts on the first term ($u = x$, $dv = \tfrac{1}{2}\sec^2(x/2)dx$, $v = \tan(x/2)$): $\int \frac{x}{2} \sec^2 \frac{x}{2} dx = x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx$.

The $\int \tan(x/2)dx$ terms cancel.

Answer: $x \tan(x/2) + C$.

OR

Evaluate $\int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$.

ANSWER

6/5

$2 \sin 2x = 4 \sin x \cos x$, so $\sqrt{(2 \sin 2x)} = 2\sqrt{(\sin x \cos x)} = 2\cos x \sqrt{(\tan x)}$.

Integrand $= \frac{1}{2\cos^4 x \sqrt{\tan x}} = \frac{(1+\tan^2 x)\sec^2 x}{2\sqrt{\tan x}}$.

Put $t = \tan x$, $dt = \sec^2 x\, dx$; limits $0 \to 1$.

$I = \frac{1}{2}\int_0^1 (t^{-1/2} + t^{3/2})dt = \frac{1}{2}\left[2\sqrt{t} + \frac{2}{5}t^{5/2}\right]_0^1 = \frac{1}{2}\left(2 + \frac{2}{5}\right) = \frac{6}{5}$.

29. Show that the lines $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew, and find the shortest distance between them. **3**

ANSWER

SD = 8/√29 units (≠ 0, so the lines are skew)

Rewrite: line 1 has $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$; line 2 has $\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$.

b_1 and b_2 are not proportional, so the lines are not parallel.

$\vec{b}_1 \times \vec{b}_2 = \hat{i}(1\cdot(-2) - (-2)\cdot 2) - \hat{j}((-1)(-2) - (-2)(1)) + \hat{k}((-1)(2) - (1)(1)) = 2\hat{i} - 4\hat{j} - 3\hat{k}$.

$\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$.

$(a_2 - a_1)\cdot(b_1 \times b_2) = 0 - 4 + 12 = 8 \neq 0$, so the lines do not intersect → skew.

$|b_1 \times b_2| = \sqrt{(4 + 16 + 9)} = \sqrt{29}$.

SD = 8/√29 units.

OR

In a cricket match the bowler, wicket-keeper and leg-slip fielder stand in a straight line at position vectors $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$. In what ratio does the wicket-keeper divide the segment from the bowler to the fielder?

ANSWER

2 : 3 (internally)

Let W divide BF in ratio k : 1. Then $\vec{W} = \frac{k\vec{F} + \vec{B}}{k+1}$.

î: (12k + 2)/(k + 1) = 6 → 12k + 2 = 6k + 6 → k = 2/3.

ĵ: (18k + 8)/(k + 1) = 12 → 6k = 4 → k = 2/3 (consistent).

Ratio = 2/3 : 1 = 2 : 3, internal.

30. X = number of students absent in a class on a Saturday has the distribution: P(X = 0) = p, P(X = 2) = 2p, P(X = 4) = 3p, P(X = 5) = p. **3**

(i) Find p. (1 mark)

(ii) Find the mean number of absent students on a Saturday. (2 marks)

ANSWER

(i) p = 1/7 (ii) mean = 3

(i) Sum of probabilities = 1: p + 2p + 3p + p = 7p = 1 → p = 1/7.

(ii) E(X) = Σ x P(x) = 0(p) + 2(2p) + 4(3p) + 5(p) = 21p = 21/7 = 3.

OR

3000 people applied for a job; two-thirds were women and the rest men. Selection was by written test. A man scores a distinction with probability 0.4 and a woman with probability 0.35. Find the probability that a randomly picked candidate has a distinction.

ANSWER

11/30 ≈ 0.367

P(F) = 2/3, P(M) = 1/3; P(D|M) = 0.4, P(D|F) = 0.35.

Total probability: P(D) = P(M)P(D|M) + P(F)P(D|F) = (1/3)(0.4) + (2/3)(0.35).

= 0.4/3 + 0.7/3 = 1.1/3 = 11/30.

(The figure 3000 is not needed.)

31. Draw the graph of y = |x + 3| and, by integration, find the area bounded by this curve and the x-axis between x = −6 and x = 0. **3**

ANSWER

Area = 9 sq units

Graph: a V shape with vertex at (−3, 0); y = −(x + 3) for x < −3, y = x + 3 for x ≥ −3.

Area = $\int_{-6}^{-3} -(x+3)dx + \int_{-3}^{0}(x+3)dx$.

First = $\left[-\frac{(x+3)^2}{2}\right]_{-6}^{-3} = 0 + \frac{9}{2} = \frac{9}{2}$.

Second = $\left[\frac{(x+3)^2}{2}\right]_{-3}^{0} = \frac{9}{2}$.

Total = 9 square units.

SECTION D

4 long answer (LA) questions of 5 marks each.

32. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$. **5**

ANSWER

dy/dx = √((1 − y²)/(1 − x²)) (proved)

Put x = sin A, y = sin B.

cos A + cos B = a(sin A − sin B).

2cos((A+B)/2)cos((A−B)/2) = 2a cos((A+B)/2) sin((A−B)/2).

Cancel: cot((A − B)/2) = a → A − B = 2cot⁻¹a (a constant).

So sin⁻¹x − sin⁻¹y = constant.

Differentiate: $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$.

Hence $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

OR

Given $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and y = sin θ, find $\frac{d^2y}{dx^2}$ at θ = π/4.

ANSWER

d²y/dx² = 2√2/a²

dx/dθ = a(−sin θ + $\frac{\frac{1}{2}\sec^2(\theta/2)}{\tan(\theta/2)}$) = a(−sin θ + 1/sin θ) = a cos²θ / sin θ.

dy/dθ = cos θ.

dy/dx = cos θ · sin θ/(a cos²θ) = (1/a) tan θ.

d²y/dx² = (1/a) sec²θ · dθ/dx = (1/a) sec²θ · sin θ/(a cos²θ) = sin θ/(a² cos⁴θ).

At θ = π/4: sin = 1/√2, cos⁴ = 1/4 → (1/√2)·4/a² = 2√2/a².

33. Find the absolute maximum and absolute minimum values of $f(x)=2x^3-15x^2+36x+1$ on [1, 5]. **5**

ANSWER

Absolute max = 56 (at x = 5); absolute min = 24 (at x = 1)

f'(x) = 6x² − 30x + 36 = 6(x − 2)(x − 3) = 0 → x = 2, 3 (both in [1, 5]).

f(1) = 2 − 15 + 36 + 1 = 24.

f(2) = 16 − 60 + 72 + 1 = 29.

f(3) = 54 − 135 + 108 + 1 = 28.

f(5) = 250 − 375 + 180 + 1 = 56.

Absolute maximum 56 at x = 5; absolute minimum 24 at x = 1.

Mistake to avoid: forgetting the end points.

34. Find the image A′ of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also write the equation of the line through A and A′. **5**

ANSWER

A′(1, 0, 7); line AA′: (x − 1)/0 = (y − 6)/(−3) = (z − 3)/2

General point on the line: P(λ, 1 + 2λ, 2 + 3λ).

$\overrightarrow{AP} = (\lambda - 1, 2\lambda - 5, 3\lambda - 1)$. For the foot, AP ⊥ (1, 2, 3):

(λ − 1) + 2(2λ − 5) + 3(3λ − 1) = 14λ − 14 = 0 → λ = 1.

Foot P = (1, 3, 5).

P is the midpoint of AA′: A′ = 2P − A = (1, 0, 7).

Direction of AA′ = (0, −6, 4) ∥ (0, −3, 2).

Line AA′: $\vec{r} = \hat{i} + 6\hat{j} + 3\hat{k} + \mu(-3\hat{j} + 2\hat{k})$, i.e. (x − 1)/0 = (y − 6)/(−3) = (z − 3)/2.

OR

On the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ find a point P which is 7 units from Q(2, 4, −1). Then write the equation of line PQ.

ANSWER

P(−4, 1, −3); PQ: (x − 2)/6 = (y − 4)/3 = (z + 1)/2

General point: P(λ − 5, 4λ − 3, −9λ + 6).

PQ² = (λ − 7)² + (4λ − 7)² + (−9λ + 7)² = 98λ² − 196λ + 147.

Set = 49: 98λ² − 196λ + 98 = 0 → (λ − 1)² = 0 → λ = 1.

P = (−4, 1, −3).

Direction Q − P = (6, 3, 2).

Line PQ: (x − 2)/6 = (y − 4)/3 = (z + 1)/2.

35. A school puts 180 students into three clubs – Sports, Music and Drama – such that: the Sports club strength equals Music + Drama together; the Music club has 20 more than half of the Sports club; and all three together have 180 students. Using the matrix method, find how many students go to each club. **5**

ANSWER

Sports 90, Music 65, Drama 25

Let Sports = x, Music = y, Drama = z.

x − y − z = 0; −x + 2y = 40 (from y = x/2 + 20); x + y + z = 180.

A = [[1, −1, −1], [−1, 2, 0], [1, 1, 1]], B = [0, 40, 180]^T.

|A| = 1(2) + 1(−1) − 1(−3) = 4 ≠ 0, so A^{-1} exists.

Cofactors give adj A = [[2, 0, 2], [1, 2, 1], [−3, −2, 1]].

X = A^{-1}B = ¼[2·0 + 0·40 + 2·180, 1·0 + 2·40 + 180, −3·0 − 2·40 + 180] = ¼[360, 260, 100].

x = 90, y = 65, z = 25.

Check: 65 + 25 = 90 ✓, 90/2 + 20 = 65 ✓, total 180 ✓.

SECTION E

3 case-study based questions of 4 marks each.

36. A company is laying a rectangular solar-panel area on a roof using 300 m of boundary material in total. A partition parallel to one pair of sides splits it into two parts. Let x m be the length of each side perpendicular to the partition and y m the length of each side parallel to it. **4**

(i) Write the equation for the total material used (boundary + partition) in terms of x and y. (1)

(ii) Express the area as a function of x. (1)

(iii) Find the critical point of the area function, confirm with the second-derivative test that it gives the maximum, and find the maximum area. (2)

ANSWER

(i) $2x + 3y = 300$ (ii) $A(x) = 100x - (2/3)x^2$ (iii) $x = 75$ m, $y = 50$ m, max area 3750 m²

(i) Two sides of length x, two of length y and the partition (length y): $2x + 3y = 300$.

(ii) $y = (300 - 2x)/3$, so $A = xy = (300x - 2x^2)/3 = 100x - (2/3)x^2$.

(iii) $A'(x) = 100 - (4/3)x = 0 \rightarrow x = 75$.

$A''(x) = -4/3 < 0 \rightarrow$ maximum at $x = 75$.

$y = (300 - 150)/3 = 50$; maximum area $= 75 \times 50 = 3750$ m².

OR

(iii) Using the first-derivative test, find the maximum area the company can enclose with the

(b) 300 m of boundary material, keeping the parallel partition.

ANSWER

Maximum area 3750 m² ($x = 75$ m, $y = 50$ m)

$A(x) = (300x - 2x^2)/3$, $A'(x) = (300 - 4x)/3$.

$A' = 0$ at $x = 75$.

For x slightly less than 75, $A' > 0$; for x slightly more, $A' < 0$ – sign changes from + to −, so $x = 75$ gives a local (and here absolute) maximum.

$y = 50$, maximum area = 3750 m².

37. A teacher writes five relations on A = {1, 2, 3}: **4**

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

(i) Which relation is reflexive and transitive but not symmetric? (1)

(ii) Which relation is reflexive and symmetric but not transitive? (1)

(iii) Which relations are symmetric but neither reflexive nor transitive? (2)

ANSWER

(i) R_4 (ii) R_5 (iii) R_1 and R_3

(i) R_4 contains (1,1), (2,2), (3,3) → reflexive; (1,2) present but (2,1) absent → not symmetric; every chain closes (e.g. (1,2),(2,2) → (1,2)) → transitive. Answer R_4.

(ii) R_5 is reflexive and symmetric, but (1,2), (2,3) ∈ R_5 while (1,3) ∉ R_5 → not transitive. Answer R_5.

(iii) R_1: symmetric; (2,2) missing → not reflexive; (2,3),(3,2) need (2,2) → not transitive.

R_3: symmetric; (2,2),(3,3) missing → not reflexive; (2,1),(1,2) need (2,2) → not transitive.

Answer: R_1 and R_3.

OR

(iii) Which ordered pairs must be added to R_2 to make it an equivalence relation?

(b)

ANSWER

(1,1), (2,2), (3,3), (2,1), (3,1), (2,3)

Reflexive: add (1,1), (2,2), (3,3).

Symmetric: for (1,2), (1,3), (3,2) add (2,1), (3,1), (2,3).

Now the relation is all of A × A (9 pairs), which is transitive too.

Pairs to add: (1,1), (2,2), (3,3), (2,1), (3,1), (2,3).

38. A bank gives loans at fixed, floating or variable interest rates; 10%, 20% and 70% of borrowers choose these respectively. The chance of defaulting is 5% for fixed-rate, 3% for floating-rate and 1% for variable-rate borrowers. **4**

(i) Find the probability that a borrower defaults. (2)

(ii) A borrower has defaulted. Find the probability that the loan was at a variable rate. (2)

ANSWER

(i) 0.018 (ii) 7/18

Let E_1, E_2, E_3 = fixed, floating, variable; D = default.

(i) $P(D) = 0.1\times0.05 + 0.2\times0.03 + 0.7\times0.01 = 0.005 + 0.006 + 0.007 = 0.018$.

(ii) Bayes: $P(E_3|D) = 0.007/0.018 = 7/18$.

— END —

Questions restated from the CBSE Class XII Board Examination 2025 (set 65/1/1) for practice. Reading passages and map items are described, not reproduced.