Roll No. ☐☐☐☐☐☐☐☐

Q.P. Code 65/1/1

MATHEMATICS

CBSE Class XII Board Examination 2024

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. The paper has 38 questions in five sections (A to E); all of them must be answered.
2. Section A: Q1–18 are multiple choice and Q19–20 are Assertion-Reason, 1 mark each.
3. Section B: Q21–25, very short answer (VSA), 2 marks each.
4. Section C: Q26–31, short answer (SA), 3 marks each.
5. Section D: Q32–35, long answer (LA), 5 marks each.
6. Section E: Q36–38, case-study based, 4 marks each.
7. There is no overall choice, but internal choices (OR) are given in 2 questions of Section B, 3 of Section C, 2 of Section D and 2 of Section E. Calculators are not allowed.

SECTION A

Q1–20 carry 1 mark each (Q1–18 MCQ; Q19–20 Assertion-Reason).

1. The function $f : \mathbb{R}_+ \to \mathbb{R}$ (where $\mathbb{R}_+$ is the set of all non-negative real numbers) is given by $f(x) = 4x + 3$. Then f is: **1**

(A) one-one but not onto **(B)** onto but not one-one

(C) both one-one and onto **(D)** neither one-one nor onto

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2. A matrix has 36 elements. How many different orders can it have? **1**

(A) 13 **(B)** 3

(C) 5 **(D)** 9

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3. Consider $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$. Which statement is true? **1**

(A) f(x) is continuous and differentiable for all x ∈ $\mathbb{R}$

(B) f(x) is continuous for all x ∈ $\mathbb{R}$

(C) f(x) is continuous and differentiable for all x ∈ $\mathbb{R}$ − {0}

(D) f(x) is discontinuous at infinitely many points

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4. f(x) is continuous on [a, b] and differentiable on (a, b). f(x) is strictly increasing on (a, b) if: **1**

(A) f′(x) < 0 for all x ∈ (a, b) **(B)** f′(x) > 0 for all x ∈ (a, b)

(C) f′(x) = 0 for all x ∈ (a, b) **(D)** f(x) > 0 for all x ∈ (a, b)

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5. Given $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, find $\frac{24}{x} + \frac{24}{y}$. **1**

(A) 7 **(B)** 6

(C) 8 **(D)** 18

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6. $\int_a^b f(x)\,dx$ equals: **1**

(A) $\int_a^b f(a - x)\,dx$

(B) $\int_a^b f(a + b - x)\,dx$

(C) $\int_a^b f(x - (a + b))\,dx$

(D) $\int_a^b f((a - x) + (b - x))\,dx$

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7. θ is the angle between two unit vectors $\hat{a}$ and $\hat{b}$, and sin θ = 3/5. Then $\hat{a} \cdot \hat{b}$ equals: **1**

(A) ± 3/5 **(B)** ± 3/4

(C) ± 4/5 **(D)** ± 4/3

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8. Find the integrating factor of $(1 - x^2)\frac{dy}{dx} + xy = ax$, where −1 < x < 1. **1**

(A) $\frac{1}{x^2 - 1}$ **(B)** $\frac{1}{\sqrt{x^2 - 1}}$

(C) $\frac{1}{1 - x^2}$ **(D)** $\frac{1}{\sqrt{1 - x^2}}$

9. A line has direction cosines √3 k, √3 k, √3 k. Find k. **1**

(A) ± 1 **(B)** ± √3

(C) ± 3 **(D)** ± 1/3

10. A linear programming problem is about optimising (maximising or minimising) a: **1**

(A) logarithmic function **(B)** linear function

(C) quadratic function **(D)** exponential function

11. If P(A | B) = P(A′ | B), which statement is true? **1**

(A) P(A) = P(A′) **(B)** P(A) = 2 P(B)

(C) P(A ∩ B) = ½ P(B) **(D)** P(A ∩ B) = 2 P(B)

12. Evaluate $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$. **1**

(A) $2x^3$ **(B)** 2

(C) 0 **(D)** $2x^3 - 2$

13. Find the derivative of $\sin(x^2)$ with respect to x at $x = \sqrt{\pi}$. **1**

(A) 1 **(B)** −1

(C) $-2\sqrt{\pi}$ **(D)** $2\sqrt{\pi}$

14. Find the order and degree of $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$. **1**

(A) 1, 2 **(B)** 2, 3

(C) 2, 1 **(D)** 2, 6

15. Find the vector whose initial point is B(3, −4, 7) and terminal point is A(2, −3, 5). **1**

(A) $\hat{i} - \hat{j} + 2\hat{k}$ **(B)** $\hat{i} + \hat{j} + 2\hat{k}$

(C) $-\hat{i} - \hat{j} - 2\hat{k}$ **(D)** $-\hat{i} + \hat{j} - 2\hat{k}$

16. The distance of the point P(a, b, c) from the y-axis is: **1**

(A) b **(B)** b^2

(C) $\sqrt{a^2 + c^2}$ **(D)** $a^2 + c^2$

17. How many corner points does the feasible region of $x \geq 0$, $y \geq 0$, $x + y \geq 4$ have? **1**

(A) 0 **(B)** 1

(C) 2 **(D)** 3

18. A and B are non-zero square matrices of the same order with $(A + B)^2 = A^2 + B^2$. Then: **1**

(A) $AB = O$ **(B)** $AB = -BA$

(C) $BA = O$ **(D)** $AB = BA$

19. Assertion (A): For $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, $\theta \in [0, 2\pi]$, we have $|A| \in [2, 4]$. **1**

Reason (R): $\cos\theta \in [-1, 1]$ for all $\theta \in [0, 2\pi]$.

(A) Both A and R are true, and R is the correct explanation of A.

(B) Both A and R are true, but R is not the correct explanation of A.

(C) A is true, but R is false.

(D) A is false, but R is true.

20. Assertion (A): No line in space can be perpendicular to all three of the x, y and z axes at the same time. **1**

Reason (R): If a line makes angles α, β, γ with the positive x, y, z axes, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

(A) Both A and R are true, and R is the correct explanation of A.

(B) Both A and R are true, but R is not the correct explanation of A.

(C) A is true, but R is false.

(D) A is false, but R is true.

SECTION B

Q21–25 are very short answer (VSA) questions, 2 marks each.

21. Is the function f(x) = x²|x| differentiable at x = 0? Justify. **2**

OR

Given $y = \sqrt{\tan \sqrt{x}}$, prove that $\sqrt{x}\,\dfrac{dy}{dx} = \dfrac{1 + y^4}{4y}$.

22. Show that f(x) = 4x³ − 18x² + 27x − 7 has no maximum or minimum value. **2**

23. Find $\int x\sqrt{1 + 2x}\,dx$. **2**

OR

Evaluate $\displaystyle\int_0^{\pi^2/4} \frac{\sin \sqrt{x}}{\sqrt{x}}\,dx$.

24. $\vec{a}$ and $\vec{b}$ are non-zero vectors with $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$. Prove that $|\vec{b}| = \sqrt{2}\,|\vec{a}|$. **2**

25. ABCD is a parallelogram (figure: AB is the top side, DC the bottom side, and the diagonal DB is drawn from D to B). Given $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, find $\vec{AD}$ and then the area of the parallelogram. **2**

SECTION C

Q26–31 are short answer (SA) questions, 3 marks each.

26. On A = {1, 2, 3, 4, 5}, the relation R = {(x, y) : |x² − y²| < 8}. Check whether R is reflexive, symmetric and transitive. **3**

OR

f : $\mathbb{R} \to \mathbb{R}$ is given by f(x) = ax + b with f(1) = 1 and f(2) = 3. Find f(x) and check whether f is one-one and onto.

27. Given $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **3**

OR

Find dy/dx if y = (tan x)^x.

28. Find $\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$. **3**

OR

Evaluate $\int_1^3 \left(|x-1| + |x-2| + |x-3|\right) dx$.

29. Find the particular solution of $x^2\frac{dy}{dx} - xy = x^2\cos^2\left(\frac{y}{2x}\right)$, given y = π/2 when x = 1. **3**

30. Solve graphically: maximise Z = 500x + 300y subject to x + 2y ≤ 12, 2x + y ≤ 12, 4x + 5y ≥ 20, x ≥ 0, y ≥ 0. **3**

31. E and F are independent events with P($\bar{E}$) = 0.6 and P(E ∪ F) = 0.6. Find P(F) and P($\bar{E} \cup \bar{F}$). **3**

SECTION D

Q32–35 are long answer (LA) questions, 5 marks each.

32. For $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$. **5**

OR

Given $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$, find $(a + x) - (b + y)$.

33. Evaluate $\displaystyle\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x}\, dx$. **5**

OR

Evaluate $\displaystyle\int_0^{\pi/2} \sin 2x \, \tan^{-1}(\sin x)\, dx$.

34. Using integration, find the area of the part of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$ that lies between the lines $x = -2$ and $x = 2$. **5**

35. The image of a point P(x, y, z) in the line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ is P′(1, 0, 7). Find P. **5**

SECTION E

Q36–38 are case-study based questions, 4 marks each.

36. *Case Study 1: Traffic police use speed cameras that can catch a speeding vehicle from up to 300 m away. One camera is fixed on a pole 5 m high, and a car C moves away from the pole at 20 m/s. When the car is x m from the foot of the pole, the angle of elevation of the camera from the car is θ.* **4**

(i) Express θ in terms of the height of the camera and x. (1)

(ii) Find dθ/dx. (1)

(iii)(a) Find the rate of change of the angle of elevation with respect to time when the car is 50 m from the pole. (2)

OR

(iii) (b) For another car 50 m from the foot of the pole, the angle of elevation is changing at the rate of 3/101 rad/s (in magnitude). Find the speed of this car. (2)

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37. *Case Study 2: Climate change has made air turbulence more common. An airplane meets severe, moderate or light turbulence with equal probability, and the chance of arriving late is 55%, 37% and 17% respectively for these three.* **4**

(i) Find the probability that an airplane reaches its destination late. (2)

(ii) Given that the airplane reached late, find the probability that this was due to moderate turbulence. (2)

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38. *Case Study 3: A function has an inverse only when it is one-one and onto. Sine on $\mathbb{R}$ is neither (its graph is the familiar wave between −1 and 1), so sine is restricted to an interval A, mapped onto [−1, 1], so that $\sin^{-1} : [-1, 1] \to A$ exists.* **4**

(i) Give an example of such an interval A other than the principal value branch. (1)

(ii) Using the principal value branch, find $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. (1)

(iii)(a) Draw the graph of $\sin^{-1}x$ on [−1, 1] for its principal value branch. (2)

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. (2)

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Questions restated from the CBSE Class XII Board Examination 2024 (set 65/1/1) for practice. Reading passages and map items are described, not reproduced.