Roll No. ☐☐☐☐☐☐☐☐

Q.P. Code 65/1/1

MATHEMATICS

CBSE Class XII Board Examination 2023 — with answers

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. The paper has 38 questions in five sections (A to E); all of them must be answered.
2. Section A: Q1–18 are multiple choice and Q19–20 are Assertion-Reason questions, 1 mark each.
3. Section B: Q21–25, very short answer (VSA), 2 marks each.
4. Section C: Q26–31, short answer (SA), 3 marks each.
5. Section D: Q32–35, long answer (LA), 5 marks each.
6. Section E: Q36–38, case-study based, 4 marks each.
7. There is no overall choice, but internal choices (OR) are given in 2 questions of Section B, 3 of Section C, 2 of Section D and 2 of Section E. Calculators are not allowed.

SECTION A

Q1–20 carry 1 mark each (MCQ; Q19–20 are Assertion-Reason).

1. A square matrix A satisfies $A^2 - 3A + I = O$. If $A^{-1} = xA + yI$, find x + y. **1**

(A) −2 **(B) 2**

(C) 3 **(D)** −3

ANSWER
(b) 2

Multiply $A^2 - 3A + I = O$ by A^{-1}: $A - 3I + A^{-1} = O$.
So $A^{-1} = -A + 3I$, i.e. x = −1, y = 3.
x + y = −1 + 3 = 2.

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2. A is a 2 × 2 matrix with |A| = 2. What is $|4A^{-1}|$? **1**

(A) 4 **(B)** 2

(C) 8 **(D)** $\frac{1}{32}$

ANSWER
(c) 8

For an n × n matrix, $|kB| = k^n|B|$. Here n = 2.
$|4A^{-1}| = 4^2 \cdot |A^{-1}| = 16 \cdot \frac{1}{|A|} = 16 \cdot \frac{1}{2} = 8$.
Common mistake: writing $4|A^{-1}|$ instead of $4^2|A^{-1}|$.

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3. For a 3 × 3 matrix A, |adj A| = 64. Then |A| equals: **1**

(A) 8 only **(B)** −8 only

(C) 64 **(D) 8 or −8**

ANSWER
(d) 8 or −8

For an n × n matrix, $|\text{adj } A| = |A|^{n-1}$. With n = 3: $|A|^2 = 64$.
So |A| = ±8 – both signs are possible.

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4. Let $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$. If 2A + B is the zero matrix, find B. **1**

(A) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

ANSWER

(b) B = [[−6, −8], [−10, −4]]

2A + B = O gives B = −2A.

$2A = \begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$, so $B = \begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$.

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5. If $\frac{d}{dx}(f(x)) = \log x$, then f(x) is: **1**

(A) $-\frac{1}{x} + C$

(B) $x(\log x - 1) + C$

(C) $x(\log x + x) + C$

(D) $\frac{1}{x} + C$

ANSWER

(b) x(log x − 1) + C

f(x) = ∫ log x dx. Integrate by parts taking log x as first function and 1 as second:

$\int \log x \, dx = x \log x - \int x \cdot \frac{1}{x} dx = x \log x - x + C = x(\log x - 1) + C.$

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6. Evaluate $\int_0^{\pi/6} \sec^2\left(x - \frac{\pi}{6}\right) dx$. **1**

(A) $\frac{1}{\sqrt{3}}$

(B) $-\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

ANSWER

(a) 1/√3

Antiderivative: tan(x − π/6).

Value = tan(0) − tan(−π/6) = 0 + 1/√3 = 1/√3.

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7. Find the sum of the order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$. **1**

(A) 5 **(B)** 2

(C) 3 **(D)** 4

ANSWER

(c) 3

Highest derivative is d^2y/dx^2 → order = 2.

It appears with power 1 and the equation is a polynomial in the derivatives → degree = 1.

Sum = 3. (The cube is on dy/dx, not the highest derivative, so it does not affect the degree.)

8. For what value of p are the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ perpendicular? **1**

(A) 3 **(B)** −3

(C) $-\frac{17}{3}$ **(D)** $\frac{17}{3}$

ANSWER

(a) p = 3

Perpendicular ⇒ dot product = 0.

2(−4) + p(−6) + 1(26) = 0 ⇒ −8 − 6p + 26 = 0 ⇒ 6p = 18 ⇒ p = 3.

9. Find the value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$. **1**

(A) 2 **(B)** 0

(C) 1 **(D) −1**

ANSWER

(d) −1

$\hat{i} \times \hat{j} = \hat{k}$, and $\hat{k} \cdot \hat{j} = 0$.

$\hat{j} \times \hat{i} = -\hat{k}$, and $-\hat{k} \cdot \hat{k} = -1$.

Total = 0 + (−1) = −1.

10. Given $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, find $|\vec{b}|$. **1**

(A) √14 **(B) 3**

(C) √12 **(D)** √17

ANSWER

(b) 3

$\vec{b} = \hat{i} - \vec{a} = -\hat{i} + 2\hat{j} - 2\hat{k}$.

$|\vec{b}| = \sqrt{1 + 4 + 4} = 3$.

11. Find the direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$. **1**

(A) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

(B) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$

(C) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$

(D) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

ANSWER

(d) 2/7, −3/7, 6/7

First write the line in standard form (coefficient of x, y, z must be +1):

$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1/2}{6}$.

Direction ratios: 2, −3, 6; magnitude √(4 + 9 + 36) = 7.

Direction cosines: 2/7, −3/7, 6/7.

Common mistake: taking 3 and 12 directly from the given form (gives option b).

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12. If P(A|B) = 0.3, P(A) = 0.4 and P(B) = 0.8, find P(B|A). **1**

(A) 0.6 **(B)** 0.3

(C) 0.06 **(D)** 0.4

ANSWER

(a) 0.6

P(A ∩ B) = P(A|B)·P(B) = 0.3 × 0.8 = 0.24.

P(B|A) = P(A ∩ B)/P(A) = 0.24/0.4 = 0.6.

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13. f(x) = 3x + 5 for x ≥ 2 and f(x) = kx² for x < 2. For which k is f continuous? **1**

(A) $-\frac{11}{4}$ **(B)** $\frac{4}{11}$

(C) 11 **(D)** $\frac{11}{4}$

ANSWER

(d) k = 11/4

Only x = 2 needs checking.

Right-hand value: f(2) = 3(2) + 5 = 11. Left-hand limit: k(2)² = 4k.

Continuity: 4k = 11 ⇒ k = 11/4.

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14. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. If $(3I + 4A)(3I - 4A) = x^2 I$, find the value(s) of x. **1**

(A) ±√7 **(B)** 0

(C) ±5 **(D)** 25

ANSWER

(c) x = ±5

I and A commute, so (3I + 4A)(3I − 4A) = 9I − 16A².

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I.$

So 9I + 16I = 25I = x²I ⇒ x² = 25 ⇒ x = ±5.

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15. Find the general solution of $x\,dy - (1 + x^2)\,dx = dx$. **1**

(A) $y = 2x + \frac{x^3}{3} + C$ **(B)** $y = 2\log x + \frac{x^3}{3} + C$

(C) $y = \frac{x^2}{2} + C$ **(D)** $y = 2\log x + \frac{x^2}{2} + C$

ANSWER

(d) y = 2 log x + x²/2 + C

Rearrange: x dy = (2 + x²) dx ⇒ dy = (2/x + x) dx.

Integrate: y = 2 log x + x²/2 + C.

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16. f(x) = a(x − cos x) is strictly decreasing on $\mathbb{R}$. Then a lies in: **1**

(A) {0} **(B)** (0, ∞)

(C) (−∞, 0) **(D)** (−∞, ∞)

ANSWER

(c) a ∈ (−∞, 0)

f'(x) = a(1 + sin x). Since 1 + sin x ≥ 0 (zero only at isolated points), f' < 0 needs a < 0.

So a ∈ (−∞, 0).

17. An LPP has a feasible region with corner points (2, 72), (15, 20) and (40, 15). For the objective function z = 18x + 9y: **1**

(A) z is maximum at (2, 72) and minimum at (15, 20)

(B) z is maximum at (15, 20) and minimum at (40, 15)

(C) z is maximum at (40, 15) and minimum at (15, 20)

(D) z is maximum at (40, 15) and minimum at (2, 72)

ANSWER

(c) max at (40, 15), min at (15, 20)

Evaluate z at each corner:

(2, 72): 36 + 648 = 684

(15, 20): 270 + 180 = 450 (least)

(40, 15): 720 + 135 = 855 (greatest)

So maximum at (40, 15), minimum at (15, 20).

18. How many corner points does the feasible region of $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ have? **1**

(A) 2 **(B)** 3

(C) 4 **(D)** 5

ANSWER

(a) 2

The region lies below y = x and below y = (x + 2)/2, above y = 0.

y = x and 2y = x + 2 meet at (2, 2); y = x passes through (0, 0).

The region is unbounded to the right (between y = 0 and the lines). Its corner points are (0, 0) and (2, 2) – two in all.

19. Assertion (A): For $x \in [-1, 1]$, the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$ has range $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$. **1**

Reason (R): The principal value branch of $\sin^{-1} x$ has range $[0, \pi]$.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

ANSWER

(c) A true, R false

$\sin^{-1}x \in [-\pi/2, \pi/2]$, so $2\sin^{-1}x \in [-\pi, \pi]$.

Adding $3\pi/2$: $f(x) \in [\pi/2, 5\pi/2]$ → A is true.

The principal range of $\sin^{-1}x$ is $[-\pi/2, \pi/2]$, not $[0, \pi]$ (that is $\cos^{-1}x$) → R is false.

20. Assertion (A): The line through (1, 2, 3) and (3, −1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$. **1**

Reason (R): The line through (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

ANSWER

(d) A false, R true

R is the standard two-point form – true.

Direction ratios: (3 − 1, −1 − 2, 3 − 3) = (2, −3, 0). So the line is (x − 3)/2 = (y + 1)/(−3) = (z − 3)/0.

The assertion has +3 under y, so A is false.

SECTION B

Q21–25 are very short answer (VSA) questions of 2 marks each.

21. A = {1, 2, 3, 4} and f : A → B, f(x) = 2x, is one-one and onto. Find B. **2**

ANSWER

B = {2, 4, 6, 8}

f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8.

Since f is onto, B must equal the range: B = {2, 4, 6, 8}.

OR

Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$.

ANSWER

5π/4

3π/4 is not in [−π/2, π/2], so write sin(3π/4) = sin(π/4): $\sin^{-1}(\sin\frac{3\pi}{4}) = \frac{\pi}{4}$.

3π/4 lies in [0, π], so $\cos^{-1}(\cos\frac{3\pi}{4}) = \frac{3\pi}{4}$.

tan⁻¹(1) = π/4.

Sum = π/4 + 3π/4 + π/4 = 5π/4.

Common mistake: writing sin⁻¹(sin 3π/4) = 3π/4.

22. Find every vector of magnitude $3\sqrt{3}$ that is collinear with $\hat{i} + \hat{j} + \hat{k}$. **2**

ANSWER

±(3î + 3ĵ + 3k̂)

Unit vector along î + ĵ + k̂: $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$.

Collinear vectors of magnitude 3√3 are ± 3√3 × unit vector = ±(3î + 3ĵ + 3k̂).

Don't forget the opposite direction – collinear includes both.

23. Points A, B, C lie on a line in that order with position vectors $\vec{a}, \vec{b}, \vec{c}$ (C is beyond B). If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$. **2**

ANSWER

c = (5b − a)/4

$\vec{AC} = \vec{c} - \vec{a}$ and $\vec{AB} = \vec{b} - \vec{a}$.
$\vec{c} - \vec{a} = \frac{5}{4}(\vec{b} - \vec{a})$
$\vec{c} = \frac{5}{4}\vec{b} - \frac{1}{4}\vec{a} = \frac{5\vec{b}-\vec{a}}{4}$.

OR

Are the lines x = 2λ + 2, y = 7λ + 1, z = −3λ − 3 and x = −μ − 2, y = 2μ + 8, z = 4μ + 5 perpendicular? Check.

ANSWER

Yes, they are perpendicular

Direction ratios: first line (2, 7, −3), second line (−1, 2, 4).
Dot product: 2(−1) + 7(2) + (−3)(4) = −2 + 14 − 12 = 0.
Since $a_1a_2 + b_1b_2 + c_1c_2 = 0$, the lines are perpendicular.

24. Given $y = \left(x + \sqrt{x^2 - 1}\right)^2$, prove that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$. **2**

ANSWER

$\sqrt{(x^2 - 1)} \cdot dy/dx = 2y$, squaring gives the result

Chain rule: $\frac{dy}{dx} = 2\left(x + \sqrt{x^2 - 1}\right)\left(1 + \frac{x}{\sqrt{x^2-1}}\right)$.
The bracket = $\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}$, so $\frac{dy}{dx} = \frac{2(x+\sqrt{x^2-1})^2}{\sqrt{x^2-1}} = \frac{2y}{\sqrt{x^2-1}}$.
Thus $\sqrt{x^2 - 1}\,\frac{dy}{dx} = 2y$. Squaring: $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

25. Prove that $f(x) = \frac{16\sin x}{4+\cos x} - x$ is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$. **2**

ANSWER

$f'(x) = \cos x(56 - \cos x)/(4 + \cos x)^2 < 0$ on $(\pi/2, \pi)$

Quotient rule: $f'(x) = \frac{16\cos x(4+\cos x)+16\sin^2 x}{(4+\cos x)^2} - 1 = \frac{64\cos x+16}{(4+\cos x)^2} - 1$.
Combine: numerator = 64 cos x + 16 − (16 + 8 cos x + $\cos^2$x) = 56 cos x − $\cos^2$x = cos x(56 − cos x).
So $f'(x) = \frac{\cos x(56-\cos x)}{(4+\cos x)^2}$.
On $(\pi/2, \pi)$: cos x < 0, while 56 − cos x > 0 and the denominator > 0. Hence f'(x) < 0, so f is strictly decreasing there.

SECTION C

Q26–31 are short answer (SA) questions of 3 marks each.

26. Evaluate $\int_0^{\pi/2} [\log(\sin x) - \log(2\cos x)]\, dx$. **3**

ANSWER

$-(\pi/2)\log 2$

Integrand = log(tan x / 2) = log tan x − log 2.

Let J = ∫₀^{π/2} log tan x dx. Using ∫₀^a f(x)dx = ∫₀^a f(a − x)dx, J = ∫₀^{π/2} log cot x dx = −J, so J = 0.

Hence I = 0 − (π/2) log 2 = −(π/2) log 2 (equivalently (π/4) log(1/4)).

27. Find $\int \frac{dx}{\sqrt{x}\,(\sqrt{x}+1)(\sqrt{x}+2)}$. **3**

ANSWER

$2\log|(\sqrt{x}+1)/(\sqrt{x}+2)| + C$

Put √x = t, so dx/(2√x) = dt, i.e. dx/√x = 2 dt.

I = ∫ 2 dt/((t + 1)(t + 2)).

Partial fractions: 1/((t + 1)(t + 2)) = 1/(t + 1) − 1/(t + 2).

I = 2[log|t + 1| − log|t + 2|] + C = 2 log|(√x + 1)/(√x + 2)| + C.

28. Solve $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$ with y(0) = 0 (particular solution). **3**

ANSWER

$y = \tan x - 1 + e^{-\tan x}$

Linear equation with P = sec²x, Q = tan x sec²x. IF = $e^{\int \sec^2 x\,dx} = e^{\tan x}$.

Solution: $ye^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x}\, dx$.

Put tan x = t: ∫ t e^t dt = e^t(t − 1). So $ye^{\tan x} = e^{\tan x}(\tan x - 1) + C$.

y(0) = 0: 0 = 1·(−1) + C ⇒ C = 1.

Particular solution: $y = \tan x - 1 + e^{-\tan x}$.

OR

Solve $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$.

ANSWER

$y + \sqrt{(x^2 + y^2)} = Cx^2$

Rewrite: $\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$ – homogeneous.

Put y = vx, dy/dx = v + x dv/dx: $v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$ (taking x > 0).

Separate: $\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$.

Integrate: $\log|v + \sqrt{1+v^2}| = \log|x| + \log C \Rightarrow v + \sqrt{(1 + v^2)} = Cx$.

Put v = y/x: $y + \sqrt{x^2+y^2} = Cx^2$.

29. Solve graphically: maximise $z = 6x + 3y$ subject to $4x + y \geq 80$, $3x + 2y \leq 150$, $x + 5y \geq 115$, $x \geq 0$, $y \geq 0$. **3**

ANSWER

Max z = 285 at (40, 15)

Draw the lines $4x + y = 80$, $3x + 2y = 150$, $x + 5y = 115$. The feasible region is the bounded triangle between them.

Corner points (solve pairs):

$4x + y = 80$ and $x + 5y = 115 \rightarrow (15, 20)$

$4x + y = 80$ and $3x + 2y = 150 \rightarrow (2, 72)$

$3x + 2y = 150$ and $x + 5y = 115 \rightarrow (40, 15)$

z values: $(15, 20) \rightarrow 150$; $(2, 72) \rightarrow 228$; $(40, 15) \rightarrow 285$.

Maximum z = 285 at x = 40, y = 15.

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30. A random variable X takes values 1, 2, 3 with probabilities k/2, k/3, k/6 respectively. **3**

(i) Find k.

(ii) Find $P(1 \leq X < 3)$.

(iii) Find the mean E(X).

ANSWER

k = 1; $P(1 \leq X < 3) = 5/6$; E(X) = 5/3

(i) Sum of probabilities = 1: $k/2 + k/3 + k/6 = k = 1$.

(ii) $P(1 \leq X < 3) = P(1) + P(2) = 1/2 + 1/3 = 5/6$.

(iii) $E(X) = 1(1/2) + 2(1/3) + 3(1/6) = 1/2 + 2/3 + 1/2 = 5/3$.

OR

A and B are independent events with $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

ANSWER

P(A) = 1/3, P(B) = 1/4 or P(A) = 3/4, P(B) = 2/3

Let P(A) = x, P(B) = y. Independence $\Rightarrow x(1 - y) = 1/4$ and $(1 - x)y = 1/6$.

Subtracting: $x - y = 1/12 \Rightarrow y = x - 1/12$.

Substitute: $x(13/12 - x) = 1/4 \Rightarrow 12x^2 - 13x + 3 = 0 \Rightarrow (3x - 1)(4x - 3) = 0 \Rightarrow x = 1/3$ or $3/4$.

Then y = 1/4 or 2/3.

Both pairs satisfy the conditions: (1/3, 1/4) and (3/4, 2/3).

31. Evaluate $\int_0^{\pi/2} e^x \sin x \, dx$. **3**

ANSWER

(e^(π/2) + 1)/2

Let I = ∫ e^x sin x dx. By parts twice: I = e^x sin x − ∫ e^x cos x dx = e^x sin x − [e^x cos x + ∫ e^x sin x dx] = e^x(sin x − cos x) − I.

So I = ½ e^x(sin x − cos x).

Limits: ½[e^{π/2}(1 − 0) − e^0(0 − 1)] = ½(e^{π/2} + 1).

OR

Find $\int \frac{dx}{\cos(x-a)\cos(x-b)}$.

ANSWER

(1/sin(a − b)) log|cos(x − a)/cos(x − b)| + C

Trick: multiply and divide by sin(a − b), and write a − b = (x − b) − (x − a).

sin[(x − b) − (x − a)] = sin(x − b)cos(x − a) − cos(x − b)sin(x − a).

Dividing by cos(x − a)cos(x − b): integrand = $\frac{1}{\sin(a-b)}[\tan(x-b) - \tan(x-a)]$.

Integrate: $\frac{1}{\sin(a-b)}[-\log|\cos(x-b)| + \log|\cos(x-a)|] + C = \frac{1}{\sin(a-b)} \log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + C.$

SECTION D

Q32–35 are long answer (LA) questions of 5 marks each.

32. On $\mathbb{R}$, define R = {(x, y) : xy is irrational}. Determine whether R is reflexive, symmetric and transitive. **5**

ANSWER

Not reflexive, symmetric, not transitive

Reflexive? Take x = 1: 1·1 = 1 is rational, so (1, 1) ∉ R. Not reflexive.
Symmetric? If (x, y) ∈ R then xy is irrational; yx = xy is also irrational, so (y, x) ∈ R. Symmetric.
Transitive? Take x = √2, y = 1, z = √2: (√2, 1) ∈ R since √2 is irrational, (1, √2) ∈ R, but √2·√2 = 2 is rational, so (√2, √2) ∉ R. Not transitive.
One counter-example is enough to disprove a property; symmetry needs a general argument.

33. Let $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$. Find $(AB)^{-1}$. **5**

ANSWER

$(AB)^{-1}$ = [[10, 7, 21], [−49, −34, −103], [17, 12, 36]]

Use $(AB)^{-1} = B^{-1}A^{-1}$.
$|A| = 1(3 - 0) - 2(-1 - 0) + (-2)(2 - 0) = 3 + 2 - 4 = 1$.
Cofactors: $C_{11} = 3$, $C_{12} = 1$, $C_{13} = 2$; $C_{21} = 2$, $C_{22} = 1$, $C_{23} = 2$; $C_{31} = 6$, $C_{32} = 2$, $C_{33} = 5$.

$$A^{-1} = \text{adj } A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}.$$

$$B^{-1}A^{-1} = \begin{bmatrix} 10 & 7 & 21 \\ -49 & -34 & -103 \\ 17 & 12 & 36 \end{bmatrix}.$$

Common mistake: multiplying in the order $A^{-1}B^{-1}$.

OR

Using the matrix method, solve: x + 2y + 3z = 6, 2x − y + z = 2, 3x + 2y − 2z = 3.

ANSWER

x = 1, y = 1, z = 1

Write AX = B with $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix}$, B = $[6, 2, 3]^T$.

$|A| = 1(2 - 2) - 2(-4 - 3) + 3(4 + 3) = 0 + 14 + 21 = 35 \neq 0$, so a unique solution exists.

$$\text{adj } A = \begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}, A^{-1} = (1/35) \text{ adj } A.$$

$X = A^{-1}B = (1/35)[0 + 20 + 15, 42 - 22 + 15, 42 + 8 - 15]^T = (1/35)[35, 35, 35]^T$.

So x = 1, y = 1, z = 1 (check: 1 + 2 + 3 = 6 ✓).

34. Find the vector and Cartesian equations of the line through (1, 2, −4) parallel to the line joining A(3, 3, −5) and B(1, 0, −11). Then find the distance between these two lines. **5**

ANSWER

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $(x-1)/2 = (y-2)/3 = (z+4)/6$; distance = $\sqrt{293}/7$ units

Direction of AB: $(1 - 3, 0 - 3, -11 + 5) = (-2, -3, -6)$, i.e. parallel to $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$.
Vector equation: $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$. Cartesian: $(x - 1)/2 = (y - 2)/3 = (z + 4)/6$.
Line AB: $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.
Parallel lines: $d = |\vec{b} \times (\vec{a_2} - \vec{a_1})|/|\vec{b}|$ with $\vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$.
$\vec{b} \times (\vec{a_2} - \vec{a_1}) = (3\cdot(-1) - 6\cdot1)\hat{i} - (2\cdot(-1) - 6\cdot2)\hat{j} + (2\cdot1 - 3\cdot2)\hat{k} = -9\hat{i} + 14\hat{j} - 4\hat{k}$; magnitude $\sqrt{(81 + 196 + 16)} = \sqrt{293}$.
$|\vec{b}| = 7$, so $d = \sqrt{293}/7$ units.

OR

Find the equation of the line through A(1, 2, 3) and B(3, 5, 9). Then find the points on this line at a distance of 14 units from B.

ANSWER

$(x-1)/2 = (y-2)/3 = (z-3)/6$; points (7, 11, 21) and (−1, −1, −3)

Direction ratios: (2, 3, 6). Line: $(x - 1)/2 = (y - 2)/3 = (z - 3)/6 = \lambda$.
General point $P(2\lambda + 1, 3\lambda + 2, 6\lambda + 3)$.
$PB^2 = (2\lambda - 2)^2 + (3\lambda - 3)^2 + (6\lambda - 6)^2 = 49(\lambda - 1)^2 = 14^2 \Rightarrow (\lambda - 1)^2 = 4 \Rightarrow \lambda = 3$ or $\lambda = -1$.
$\lambda = 3$: (7, 11, 21); $\lambda = -1$: (−1, −1, −3).

35. Using integration, find the area of the region bounded by the parabola $x^2 = y$, the line $y = x + 2$ and the x-axis. **5**

ANSWER

5/6 sq. units

Intersections of $y = x^2$ and $y = x + 2$: $x^2 - x - 2 = 0 \Rightarrow x = -1, 2$. The line meets the x-axis at (−2, 0).
The region enclosed by all three curves lies above the x-axis, with the line on the left (from $x = -2$ to -1) and the parabola on the right (from $x = -1$ to 0).
Area = $\int_{-2}^{-1} (x + 2)\,dx + \int_{-1}^{0} x^2\,dx = [(x + 2)^2/2]_{-2}^{-1} + [x^3/3]_{-1}^{0} = 1/2 + 1/3 = 5/6$ sq. units.
Draw the figure first – it tells you which curve bounds the region on each interval.

SECTION E

Q36–38 are case-study based questions of 4 marks each.

36. In a society, people practise three kinds of Yoga A, B and C. A Venn diagram gives these probabilities: only A = 0.32, A and B only = 0.09, only B = y, B and C only = x, only C = 0.21, none = 0.11 (A and C do not overlap). Also P(C) = 0.44. **4**

(i) Find x.

(ii) Find y.

(iii) Find P(C | B).

ANSWER

x = 0.23, y = 0.04, P(C|B) = 23/36

(i) P(C) = x + 0.21 = 0.44 ⇒ x = 0.23.

(ii) Total = 1: 0.32 + 0.09 + y + 0.23 + 0.21 + 0.11 = 1 ⇒ 0.96 + y = 1 ⇒ y = 0.04.

(iii) P(B) = 0.09 + 0.04 + 0.23 = 0.36; P(C ∩ B) = x = 0.23.

P(C|B) = 0.23/0.36 = 23/36.

OR

(iii) Find the probability that a member of the society chosen at random does Yoga of type A or B but not C. (b)

ANSWER

0.45

From the diagram x = 0.23 and y = 0.04.

A or B but not C = only A + (A and B only) + only B = 0.32 + 0.09 + 0.04 = 0.45.

37. A tank has a cylindrical top and a conical bottom (semi-vertical angle 45°). The conical part is full of water, which drips out of a tap at its bottom at a steady 2 cm³/s. **4**

(i) Write the volume of water in the cone in terms of its radius r.

(ii) Find the rate of change of the radius when $r = 2\sqrt{2}$ cm.

(iii) Find the rate at which the wet (curved) surface of the cone is decreasing when $r = 2\sqrt{2}$ cm.

ANSWER

$V = \pi r^3/3$; $dr/dt = -1/(4\pi)$ cm/s; wet surface decreases at 2 cm²/s

(i) Semi-vertical angle 45° ⇒ r = h. $V = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi r^3$.

(ii) $dV/dt = \pi r^2\, dr/dt = -2$. At $r = 2\sqrt{2}$, $r^2 = 8$: $dr/dt = -2/(8\pi) = -1/(4\pi)$ cm/s.

(iii) Slant height $l = \sqrt{2}\, r$, so wet surface $S = \pi r l = \sqrt{2}\pi r^2$.

$dS/dt = 2\sqrt{2}\pi r\, dr/dt = 2\sqrt{2}\pi(2\sqrt{2})(-1/(4\pi)) = -2$ cm²/s, i.e. decreasing at 2 cm²/s.

OR

(iii) Find the rate of change of the height h of the water at the instant when the slant height is 4 cm.

(b)

ANSWER

$dh/dt = -1/(4\pi)$ cm/s

With 45°, r = h and $l^2 = r^2 + h^2 = 2h^2$. $l = 4 \Rightarrow h = r = 2\sqrt{2}$ cm.

$V = \tfrac{1}{3}\pi h^3 \Rightarrow dV/dt = \pi h^2\, dh/dt = -2 \Rightarrow dh/dt = -2/(8\pi) = -1/(4\pi)$ cm/s (height falls at $1/(4\pi)$ cm/s).

38. A roller-coaster track follows $f(x) = a(x + 9)(x + 1)(x - 3)$ and cuts the y-axis at $(0, -1)$. **4**

(i) Find a.

(ii) Find $f''(1)$.

ANSWER

$a = 1/27$; $f''(1) = 20/27$

(i) $f(0) = a(9)(1)(-3) = -27a = -1 \Rightarrow a = 1/27$.

(ii) Expand: $(x + 9)(x + 1)(x - 3) = x^3 + 7x^2 - 21x - 27$.

$f(x) = (x^3 + 7x^2 - 21x - 27)/27$, $f'(x) = (3x^2 + 14x - 21)/27$, $f''(x) = (6x + 14)/27$.

$f''(1) = 20/27$.

— END —

Questions restated from the CBSE Class XII Board Examination 2023 (set 65/1/1) for practice. Reading passages and map items are described, not reproduced.