Roll No. ☐☐☐☐☐☐☐☐

Q.P. Code 65/1/1

MATHEMATICS

CBSE Class XII Board Examination 2023

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. The paper has 38 questions in five sections (A to E); all of them must be answered.
2. Section A: Q1–18 are multiple choice and Q19–20 are Assertion-Reason questions, 1 mark each.
3. Section B: Q21–25, very short answer (VSA), 2 marks each.
4. Section C: Q26–31, short answer (SA), 3 marks each.
5. Section D: Q32–35, long answer (LA), 5 marks each.
6. Section E: Q36–38, case-study based, 4 marks each.
7. There is no overall choice, but internal choices (OR) are given in 2 questions of Section B, 3 of Section C, 2 of Section D and 2 of Section E. Calculators are not allowed.

SECTION A

Q1–20 carry 1 mark each (MCQ; Q19–20 are Assertion-Reason).

1. A square matrix A satisfies $A^2 - 3A + I = O$. If $A^{-1} = xA + yI$, find x + y. **1**

(A) −2 **(B)** 2

(C) 3 **(D)** −3

2. A is a 2 × 2 matrix with |A| = 2. What is $|4A^{-1}|$? **1**

(A) 4 **(B)** 2

(C) 8 **(D)** $\frac{1}{32}$

3. For a 3 × 3 matrix A, |adj A| = 64. Then |A| equals: **1**

(A) 8 only **(B)** −8 only

(C) 64 **(D)** 8 or −8

4. Let $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$. If 2A + B is the zero matrix, find B. **1**

(A) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

5. If $\frac{d}{dx}(f(x)) = \log x$, then f(x) is: **1**

(A) $-\frac{1}{x} + C$ **(B)** $x(\log x - 1) + C$

(C) $x(\log x + x) + C$ **(D)** $\frac{1}{x} + C$

6. Evaluate $\int_0^{\pi/6} \sec^2\left(x - \frac{\pi}{6}\right) dx$. **1**

(A) $\frac{1}{\sqrt{3}}$ **(B)** $-\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$ **(D)** $-\sqrt{3}$

7. Find the sum of the order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$. **1**

(A) 5 **(B)** 2

(C) 3 **(D)** 4

8. For what value of p are the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ perpendicular? **1**

(A) 3 **(B)** −3

(C) $-\frac{17}{3}$ **(D)** $\frac{17}{3}$

9. Find the value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$. **1**

(A) 2 **(B)** 0

(C) 1 **(D)** −1

10. Given $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, find $|\vec{b}|$. **1**

(A) √14 **(B)** 3

(C) √12 **(D)** √17

11. Find the direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$. **1**

(A) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

(B) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$

(C) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$

(D) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

12. If P(A|B) = 0.3, P(A) = 0.4 and P(B) = 0.8, find P(B|A). **1**

(A) 0.6 **(B)** 0.3

(C) 0.06 **(D)** 0.4

13. f(x) = 3x + 5 for x ≥ 2 and f(x) = kx² for x < 2. For which k is f continuous? **1**

(A) $-\frac{11}{4}$ **(B)** $\frac{4}{11}$

(C) 11 **(D)** $\frac{11}{4}$

14. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. If $(3I + 4A)(3I - 4A) = x^2 I$, find the value(s) of x. **1**

(A) ±√7 **(B)** 0

(C) ±5 **(D)** 25

15. Find the general solution of $x\,dy - (1 + x^2)\,dx = dx$. **1**

(A) $y = 2x + \frac{x^3}{3} + C$ **(B)** $y = 2\log x + \frac{x^3}{3} + C$

(C) $y = \frac{x^2}{2} + C$ **(D)** $y = 2\log x + \frac{x^2}{2} + C$

16. f(x) = a(x − cos x) is strictly decreasing on $\mathbb{R}$. Then a lies in: **1**

(A) {0} **(B)** (0, ∞)

(C) (−∞, 0) **(D)** (−∞, ∞)

17. An LPP has a feasible region with corner points (2, 72), (15, 20) and (40, 15). For the objective function z = 18x + 9y: **1**

(A) z is maximum at (2, 72) and minimum at (15, 20)

(B) z is maximum at (15, 20) and minimum at (40, 15)

(C) z is maximum at (40, 15) and minimum at (15, 20)

(D) z is maximum at (40, 15) and minimum at (2, 72)

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18. How many corner points does the feasible region of $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ have? **1**

(A) 2 **(B)** 3

(C) 4 **(D)** 5

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19. Assertion (A): For $x \in [-1, 1]$, the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$ has range $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$. **1**
Reason (R): The principal value branch of $\sin^{-1} x$ has range $[0, \pi]$.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

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20. Assertion (A): The line through (1, 2, 3) and (3, −1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$. **1**
Reason (R): The line through (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

SECTION B

Q21–25 are very short answer (VSA) questions of 2 marks each.

21. A = {1, 2, 3, 4} and f : A → B, f(x) = 2x, is one-one and onto. Find B. **2**

OR

Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$.

22. Find every vector of magnitude $3\sqrt{3}$ that is collinear with $\hat{i} + \hat{j} + \hat{k}$. **2**

23. Points A, B, C lie on a line in that order with position vectors $\vec{a}, \vec{b}, \vec{c}$ (C is beyond B). If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$. **2**

OR

Are the lines x = 2λ + 2, y = 7λ + 1, z = −3λ − 3 and x = −μ − 2, y = 2μ + 8, z = 4μ + 5 perpendicular? Check.

24. Given $y = \left(x + \sqrt{x^2 - 1}\right)^2$, prove that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$. **2**

25. Prove that $f(x) = \frac{16\sin x}{4 + \cos x} - x$ is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$. **2**

SECTION C

Q26–31 are short answer (SA) questions of 3 marks each.

26. Evaluate $\int_0^{\pi/2} [\log(\sin x) - \log(2\cos x)]\, dx$. **3**

27. Find $\int \frac{dx}{\sqrt{x}\,(\sqrt{x}+1)(\sqrt{x}+2)}$. **3**

28. Solve $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$ with y(0) = 0 (particular solution). **3**

OR

Solve $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$.

29. Solve graphically: maximise z = 6x + 3y subject to 4x + y ≥ 80, 3x + 2y ≤ 150, x + 5y ≥ 115, x ≥ 0, y ≥ 0. **3**

30. A random variable X takes values 1, 2, 3 with probabilities k/2, k/3, k/6 respectively. **3**

(i) Find k.

(ii) Find P(1 ≤ X < 3).

(iii) Find the mean E(X).

OR

A and B are independent events with $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

31. Evaluate $\int_0^{\pi/2} e^x \sin x\, dx$. **3**

OR

Find $\int \frac{dx}{\cos(x-a)\cos(x-b)}$.

SECTION D

Q32–35 are long answer (LA) questions of 5 marks each.

32. On $\mathbb{R}$, define R = {(x, y) : xy is irrational}. Determine whether R is reflexive, symmetric and transitive. **5**

33. Let $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$. Find $(AB)^{-1}$. **5**

OR

Using the matrix method, solve: x + 2y + 3z = 6, 2x − y + z = 2, 3x + 2y − 2z = 3.

34. Find the vector and Cartesian equations of the line through (1, 2, −4) parallel to the line joining A(3, 3, −5) and B(1, 0, −11). Then find the distance between these two lines. **5**

OR

Find the equation of the line through A(1, 2, 3) and B(3, 5, 9). Then find the points on this line at a distance of 14 units from B.

35. Using integration, find the area of the region bounded by the parabola $x^2 = y$, the line y = x + 2 and the x-axis. **5**

SECTION E

Q36–38 are case-study based questions of 4 marks each.

36. In a society, people practise three kinds of Yoga A, B and C. A Venn diagram gives these probabilities: only A = 0.32, A and B only = 0.09, only B = y, B and C only = x, only C = 0.21, none = 0.11 (A and C do not overlap). Also P(C) = 0.44. **4**

(i) Find x.

(ii) Find y.

(iii) Find P(C | B).

OR

(iii) (b) Find the probability that a member of the society chosen at random does Yoga of type A or B but not C.

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37. A tank has a cylindrical top and a conical bottom (semi-vertical angle 45°). The conical part is full of water, which drips out of a tap at its bottom at a steady 2 cm^3/s. **4**

(i) Write the volume of water in the cone in terms of its radius r.

(ii) Find the rate of change of the radius when $r = 2\sqrt{2}$ cm.

(iii) Find the rate at which the wet (curved) surface of the cone is decreasing when $r = 2\sqrt{2}$ cm.

OR

(iii) (b) Find the rate of change of the height h of the water at the instant when the slant height is 4 cm.

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38. A roller-coaster track follows $f(x) = a(x + 9)(x + 1)(x - 3)$ and cuts the y-axis at $(0, -1)$. **4**

(i) Find a.

(ii) Find $f''(1)$.

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Questions restated from the CBSE Class XII Board Examination 2023 (set 65/1/1) for practice. Reading passages and map items are described, not reproduced.