Roll No. ☐☐☐☐☐☐☐☐

Q.P. Code 65/2/1

MATHEMATICS (TERM II)

CBSE Class XII Board Examination 2022 (Term 2) — with answers

Time allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The paper has 14 questions in three sections: A, B and C. All sections are compulsory.
2. Section A: 6 short answer (type I) questions of 2 marks each.
3. Section B: 4 short answer (type II) questions of 3 marks each.
4. Section C: 4 long answer questions of 4 marks each.
5. Some questions offer an internal choice; attempt only one of the alternatives.
6. Question 14 is case-study based, with 2 sub-parts of 2 marks each.

SECTION A

Q1–6 carry 2 marks each.

1. For the differential equation $\left[\frac{d}{dx}(xy^2)\right] \cdot \frac{dy}{dx} + y = 0$, find the product of its order and its degree. **2**

ANSWER

Order × degree = 1 × 2 = 2

First expand the bracket: $\frac{d}{dx}(xy^2) = y^2 + 2xy\frac{dy}{dx}$.

So the equation is $y^2\frac{dy}{dx} + 2xy\left(\frac{dy}{dx}\right)^2 + y = 0$.

Highest derivative present is $dy/dx \rightarrow$ order = 1.

It is a polynomial in dy/dx and the highest power of dy/dx is 2 → degree = 2.

Product = 1 × 2 = 2.

Common mistake: reading the degree as 1 without expanding the bracket – always simplify first.

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2. Find $\int \frac{\sin 3x}{\sin x}\,dx$. **2**

ANSWER

x + sin 2x + C

Use $\sin 3x = 3\sin x - 4\sin^3 x$, so $\frac{\sin 3x}{\sin x} = 3 - 4\sin^2 x$.

Write $4\sin^2 x = 2(1 - \cos 2x)$: integrand = $3 - 2 + 2\cos 2x = 1 + 2\cos 2x$.

$\int (1 + 2\cos 2x)\,dx = x + \sin 2x + C$.

OR

Evaluate $\int_0^{\frac{1}{2}\log 3} \frac{e^x}{e^{2x}+1}\,dx$.

ANSWER

π/12

Put $t = e^x$, $dt = e^x dx$.

Limits: x = 0 → t = 1; x = ½ log 3 → t = $e^{\log\sqrt{3}} = \sqrt{3}$.

Integral = $\int_1^{\sqrt{3}} \frac{dt}{1+t^2} = [\tan^{-1} t]_1^{\sqrt{3}} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.

Tip: remember to change the limits when you substitute.

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3. $\vec{a}$ and $\vec{b}$ are unit vectors with $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$. **2**

ANSWER

π/2 (the vectors are perpendicular)

Square both sides: $4|\vec{a}|^2 + 12\,\vec{a}\cdot\vec{b} + 9|\vec{b}|^2 = 9|\vec{a}|^2 - 12\,\vec{a}\cdot\vec{b} + 4|\vec{b}|^2$.

With $|\vec{a}| = |\vec{b}| = 1$: 4 + 12 a·b + 9 = 9 − 12 a·b + 4 → 24 a·b = 0 → a·b = 0.

So cos θ = 0, θ = π/2.

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4. Two dice are thrown together and we are told the sum of the two numbers is even. What is the probability that at least one die shows 3? **2**

ANSWER

5/18

Let B = sum is even, A = at least one die shows 3.

Sum even means both odd or both even: 3×3 + 3×3 = 18 outcomes, P(B) = 18/36.

A ∩ B (at least one 3 and sum even ⇒ other die odd): (3,1), (3,3), (3,5), (1,3), (5,3) → 5 outcomes, P(A∩B) = 5/36.

P(A|B) = (5/36)/(18/36) = 5/18.

5. A can solve a particular problem with probability 2/3 and B with probability 3/5. Working independently, both attempt it. Find the probability that the problem gets solved. **2**

ANSWER

13/15

Problem is solved unless both fail.

P(A fails) = 1/3, P(B fails) = 2/5.

P(both fail) = (1/3)(2/5) = 2/15 (independence).

P(solved) = 1 − 2/15 = 13/15.

Common mistake: adding 2/3 + 3/5 – that exceeds 1.

6. Line PQ joins P(2, 2, 1) and Q(5, 1, −2). Write its cartesian equation, and then find the y-coordinate of the point on PQ whose z-coordinate is −2. **2**

ANSWER

(x−2)/3 = (y−2)/(−1) = (z−1)/(−3); y = 1

Direction ratios of PQ: (5−2, 1−2, −2−1) = (3, −1, −3).

Equation through P: $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$.

Put z = −2: (−2−1)/(−3) = 1, so (y−2)/(−1) = 1 → y = 1.

(Indeed this point is Q itself, (5, 1, −2).)

SECTION B

Q7–10 carry 3 marks each.

7. In parallelogram ABCD the diagonals are $\vec{AC} = \hat{i} + \hat{j}$ and $\vec{BD} = 2\hat{i} + \hat{j} + \hat{k}$. Find $\vec{AB}$ and $\vec{AD}$, and the area of ABCD. **3**

ANSWER

AB = −½ i − ½ k, AD = (3/2) i + j + ½ k, area = √3/2 sq. units

Let $\vec{AB} = \vec{a}$, $\vec{AD} = \vec{b}$. Then $\vec{AC} = \vec{a} + \vec{b}$ and $\vec{BD} = \vec{b} - \vec{a}$.

Adding: $2\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k} \rightarrow \vec{AD} = \frac{3}{2}\hat{i} + \hat{j} + \frac{1}{2}\hat{k}$.

Subtracting: $2\vec{a} = -\hat{i} - \hat{k} \rightarrow \vec{AB} = -\frac{1}{2}\hat{i} - \frac{1}{2}\hat{k}$.

Area = ½|AC × BD|. $(\hat{i} + \hat{j}) \times (2\hat{i} + \hat{j} + \hat{k}) = \hat{i} - \hat{j} - \hat{k}$, magnitude √3.

Area = √3/2 sq. units.

Note: with diagonals the factor is ½; with adjacent sides it is |a × b| directly.

8. Evaluate $\int_0^1 \tan^{-1} x \, dx$. **3**

ANSWER

π/4 − ½ log 2

Integrate by parts with u = $\tan^{-1}$x, dv = dx:

$\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2} dx = x \tan^{-1} x - \frac{1}{2}\log(1+x^2)$.

From 0 to 1: $(1 \cdot \frac{\pi}{4} - \frac{1}{2}\log 2) - 0 = \frac{\pi}{4} - \frac{1}{2}\log 2$.

OR

Find $\int \frac{2x}{x^2 + 3x + 2} \, dx$.

ANSWER

−2 log|x+1| + 4 log|x+2| + C

Factor: $x^2 + 3x + 2 = (x+1)(x+2)$.

$\frac{2x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$: 2x = A(x+2) + B(x+1).

x = −1: −2 = A → A = −2. x = −2: −4 = −B → B = 4.

Integral = $-2\log|x+1| + 4\log|x+2| + C$.

9. Find the particular solution of the differential equation $(y + 3x^2)\frac{dx}{dy} = x$, given that y = 1 when x = 1. **3**

ANSWER

$y = 3x^2 - 2x$

Flip: $\frac{dy}{dx} = \frac{y + 3x^2}{x} \rightarrow \frac{dy}{dx} - \frac{y}{x} = 3x$ (linear, P = −1/x, Q = 3x).

I.F. = $e^{\int -1/x\,dx} = e^{-\log x} = 1/x$.

Solution: $y \cdot \frac{1}{x} = \int 3x \cdot \frac{1}{x} dx = 3x + C \rightarrow y = 3x^2 + Cx$.

y(1) = 1: 1 = 3 + C → C = −2.

Particular solution: $y = 3x^2 - 2x$.

Careful: the given equation has dx/dy, not dy/dx.

10. Find the equation of the plane through (2, 1, 0), (3, −2, −2) and (1, 1, −7), and its distance from the origin. **3**

ANSWER

$7x + 3y - z = 17$; distance $17/\sqrt{59}$

Take A(2,1,0). AB = (1, −3, −2), AC = (−1, 0, −7).

Normal n = AB × AC = ((−3)(−7) − (−2)(0), (−2)(−1) − (1)(−7), (1)(0) − (−3)(−1)) = (21, 9, −3) ∥ (7, 3, −1).

Plane: 7(x−2) + 3(y−1) − (z−0) = 0 → 7x + 3y − z = 17.

Distance from O = |−17|/√(49+9+1) = 17/√59.

Check: (3,−2,−2): 21 − 6 + 2 = 17 ✓.

OR

Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x + 1 = \frac{y+2}{2} = \frac{z-1}{3}$.

ANSWER

$3\sqrt{21}/7$ units (= $\sqrt{(27/7)}$)

Both lines have direction b = (1, 2, 3) → parallel.

Points: $a_1 = (0, 1, 2)$, $a_2 = (-1, -2, 1)$; $a_2 - a_1 = (-1, -3, -1)$.

For parallel lines $d = |(a_2 - a_1) \times b| / |b|$.

(−1,−3,−1) × (1,2,3) = ((−3)(3) − (−1)(2), (−1)(1) − (−1)(3), (−1)(2) − (−3)(1)) = (−7, 2, 1), magnitude √54.

|b| = √14. d = √(54/14) = √(27/7) = $3\sqrt{21}/7$.

Mistake to avoid: the skew-line formula (with $b_1 \times b_2$) fails here because $b_1 \times b_2 = 0$.

SECTION C

Q11–14 carry 4 marks each. Q14 is a case-study based question.

11. The line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ meets the plane x − y + z = 5 at a point. Find the distance of this point from (−1, −5, −10). **4**

ANSWER

13 units

General point on the line: (3λ + 2, 4λ − 1, 12λ + 2).

Put in the plane: (3λ+2) − (4λ−1) + (12λ+2) = 5 → 11λ + 5 = 5 → λ = 0.

Intersection point: (2, −1, 2).

Distance from (−1, −5, −10): √(3² + 4² + 12²) = √169 = 13.

12. Evaluate $\int_0^1 x(1-x)^n\,dx$. **4**

ANSWER

1/((n+1)(n+2))

Use $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ with a = 1:

I = $\int_0^1 (1-x)\,x^n\,dx = \int_0^1 (x^n - x^{n+1})dx$.

= $\left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2}$.

= $\frac{1}{(n+1)(n+2)}$.

13. Using integration, find the area of the smaller region cut off from the circle $4x^2 + 4y^2 = 9$ by the line $2x + 2y = 3$. **4**

ANSWER

$(9/16)(\pi - 2) = 9\pi/16 - 9/8$ sq. units

Circle: $x^2 + y^2 = 9/4$ (radius 3/2). Line: $x + y = 3/2$, meets the circle at (3/2, 0) and (0, 3/2).
Smaller region lies in the first quadrant between the line and the arc.

Area $= \int_0^{3/2} \sqrt{\frac{9}{4} - x^2}\, dx - \int_0^{3/2} \left(\frac{3}{2} - x\right) dx.$

First integral $= \left[\frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\right]_0^{3/2} = \frac{9}{8}\cdot\frac{\pi}{2} = \frac{9\pi}{16}.$

Second integral $= [\frac{3}{2}x - \frac{x^2}{2}]_0^{3/2} = \frac{9}{4} - \frac{9}{8} = \frac{9}{8}.$

Area $= 9\pi/16 - 9/8 = (9/16)(\pi - 2)$ sq. units.

OR

The region bounded by the parabola $y^2 = 4ax$ and the line $x = 4a$ has area 256/3 square units. Using integration, find a ($a > 0$).

ANSWER

$a = 2$

The region is symmetric about the x-axis.

Area $= 2\int_0^{4a} 2\sqrt{a}\sqrt{x}\, dx = 4\sqrt{a}\cdot\frac{2}{3}[x^{3/2}]_0^{4a} = \frac{8}{3}\sqrt{a}\,(8a^{3/2}) = \frac{64a^2}{3}.$

Set $64a^2/3 = 256/3 \rightarrow a^2 = 4 \rightarrow a = 2$ (since $a > 0$).

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14. At the start of a cricket match a fair coin (head and tail equally likely) is tossed, and the team that wins the toss chooses to bat or bowl. Answer the following: **4**

(a) If this coin is tossed twice, find the probability distribution of the number of tails.

(b) Find the probability of getting at least one head when the coin is tossed three times.

ANSWER

(a) X = 0, 1, 2 with P = 1/4, 1/2, 1/4; (b) 7/8

(a) Outcomes HH, HT, TH, TT, each 1/4. X = number of tails.
P(X=0) = P(HH) = 1/4; P(X=1) = P(HT, TH) = 1/2; P(X=2) = P(TT) = 1/4. (Sum = 1 ✓)
(b) 8 equally likely outcomes; 'no head' is only TTT.
P(at least one head) = 1 − 1/8 = 7/8.

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Questions restated from the CBSE Class XII Board Examination 2022 (set 65/2/1) for practice. Reading passages and map items are described, not reproduced.